Chapter 1

Robotic Grasping and Manipulation

In this chapter, we consider problems that arise in designing, building, planning, and controlling operations of robotic hands and end–effectors. The purpose of such devices is often manifold, and it typically includes grasping and fine manipulation of ojects in an accurate, delicate yet firm way. We survey the state-of-the-art reached by scientific research and literature about the problems engendered by these often conflicting requirements, and the work that has been done in this area over the last two decades. Because of space limitations, the chapter does not attempt at providing a survey of the technology of robot hands, but rather it is oriented towards covering the theoretical framework, analytical results, and open problems in robotic manipulation.

1.1 Introduction

In many roboticists, the admiration for what nature accomplishes in everiday's functions of human beings and animals is the original stimulus for their research in emulating these capabilities in artificial life. Among the many awesome realizations of nature, few of the human abilities distinguish man from animals as deeply as manipulation and speech. Indeed, there are animals that can see, hear, walk, swim, etc. more effcicently than men - but language and manipulation skills are peculiar of our race, and constitute a continuing source of amazement for scientists. In this chapter, we will consider in detail the implementation of artificial systems to replicate in part the manipulating ability of the human hand.

The three most important functions of the human hand are to explore, to restrain, and to precisely move objects. The first function falls within the realm of *haptics*, an active research area in its own merits [46]. We will not attempt an exhaustive coverage of this area. The work in robot hands has mostly tried to understand and to emulate the other two functions. We will distinguish

Figure 1.1: The University of Bologna dextrous hand [64]

between the task of restraining objects, sometimes called *grasping* or *fixturing*, and the task of manipulating objects with fingers (in contrast to manipulation with the robot arm), sometimes called *dexterous manipulation*.

While grippers and fixtures have been used extensively in industry, one can argue that the field of robot grasping started with the work of Asada and Hanafusa [4] and Salisbury's first three-fingered robotic hand [61]. Since then, many hand designs have been proposed, ranging from rather simple devices to very sophisticated multifingered hands such as the Utah-MIT hand [41]. Extensive surveys on robot hand systems are for instance those reported in [31, 37, 71, 93], and more recently [1, 75, 7].

In robot hand design, it can be observed that there are two prevailing philosophies, which can be identified with an *anthropomorphic* vs. a *minimalistic* approach to design. While the former philosophy basically attempts at replicating the human hand capabilities by imitating its mechanical structure, the latter focuses on realization of some desirable grasping or manipulation features by purposeful design of mechanisms that have no intentional resemblance with any biological system. In the latter group, there have been a number of efforts focussing on reduced-complexity multifingered hands. Two examples of robot hands inspired to the two approaches and developed by groups participating in the RAMSETE project are reported in figures 1.1 and 1.2, respectively

Design of robot hands still poses many challenges to the research community, and several are common to the two approaches above. However, it seems fair (though perhaps slightly oversimplifying) to affirm that anthropomorphic design is mostly confronted with technological problems such as accuracy and miniaturization of sensors and actuators, power and signal transmission, etc.. In minimalistic design, instead, the emphasis of current research is more on the theoretical analysis of manipulation systems, and their deep understanding in order to allow full exploitation of limited hardware capabilities. This chapter

Figure 1.2: The University of Pisa dexterous gripper

is more focussed on the latter class of problems.

Hardware complexity reduction can be achieved in several ways. For instance, when grasp robustness is considered, it can be observed that enveloping grasps are superior in terms of restraining objects. Enveloping grasps [101], in contrast to fingertip grasps, are formed by wrapping the fingers (and the palm) around the object. Indeed, this is easily seen also in human grasping, where fingertips and distal phalanges are used in fingertip grasps for fine manipulation, while the inner parts of the hand (palm and proximal phalanges) are used in enveloping grasps for restraint [20, 40]). One of the first attempts at realizing a reduced-complexity gripper was a three fingered hand powered by four actuators [103] that was designed to grasp by enveloping. Variations of this basic theme are also seen in grippers designed for the so called *whole arm grasps* [89] and *power grasps* [67]. On the other hand, for achieving dextrous manipulation with a simplified hardware, the purposeful introduction of nonholonomic phenomena in manipulation by rolling has been advocated, and experimentally demonstrated, by several authors (see e.g. [19, 53, 71, 10]). Different modalities of manipulation and grasping share some fundamental theoretical framework, analytical results, and open problems, that are the subject of this chapter's survey.

1.2 Kinematics of Manipulation

The model of the hand we assume is comprised of an arbitrary number of fingers (i.e. simple chains of links -*phalanges* -, connected through rotoidal or prismatic joints), and of an object, which is in contact with some or all of the phalanges We let $\mathbf{q}$ denote a vector of generalized coordinates, completely describing the configuration of the fingers; and $\mathbf{u} = (\mathbf{p}_o, \mathbf{R}_o) \in SE(3)$ denote the configuration (position and orientation) of the object. With a slight abuse of notation, we also denote with $\dot{\mathbf{q}}$ and $\dot{\mathbf{u}}$ the elements of the tangent space to these configuration spaces (hence $\dot{\mathbf{u}} \in se(3)$ is the object twist).

Contacts represent a particular kind of kinematic constraint on the allowable configurations of the system, and cause most of the differences in the analysis of dextrous manipulation from other robotic systems. Contact constraints are typically unilateral, non-holonomic constraints on the generalized coordinates system, written in general in the form

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, \dot{\mathbf{u}}) \geq 0. \tag{1.1}$$

The inequality relationship reflects the fact that contact can be lost if the contacting bodies are brought away from each other. This involves an abrupt change of the structure of the model under consideration. To avoid analytical difficulties, it is usually assumed that manipulation is studied during time intervals when constraints hold with the equal sign (this is not the case in the study of grasping, where the study of these inequalities is crucial to understanding closure properties). The constraint relationship (1.1) is not in general integrable, i.e., it cannot be expressed in terms of $\mathbf{q}$ and $\mathbf{u}$ only: integrable constraints are called "holonomic". Holonomic constraints between generalized coordinates reduce the number of independent coordinates necessary to describe the system configuration (degrees of freedom), and can be assumed to be removed from the description of the system by proper coordinate substitution. Nonholonomic constraints, on the contrary, do not reduce the number of degrees-of-freedom of the system, but rather reduce the number of independent coordinate velocities.

Contact kinematics is a study of the relationship between the location of the point of contact as a function of the relative motion of two contacting bodies. The first fundamental work in this area is due to Cai and Roth [16], who studied rigid planar bodies in point contact. They derived a relationship for the rates of change of the location of the point of contact as a function of the angular and linear velocities and accelerations of the contacting bodies. Montana [69] provided a more formal description of the configuration space associated with two contacting bodies, and derived the equations of kinematic contact that relate the time derivatives of contact coordinates with the relative angular and linear velocities. These equations include terms that depend on the curvature of the contacting bodies. Sarkar, Kumar, and Yun [90], extended this work to include acceleration terms. By using intrinsic geometric properties for the contacting surfaces, they showed the explicit dependence on the Christoffel symbols and their time derivatives. This set of results is directly relevant to dexterous manipulation [75], to the analysis of higher order closure properties [86], to stability analysis [36], and to manipulability by rolling [58].

To describe in more detail contact constraints that are in effect in dextrous manipulation systems, consider a contact between the i-th phalanx and the object, occurring at time t at a point described in an inertial base frame B by the vector $\mathbf{x}_i$. A generic point on the surface of the phalanx will be described, in a frame C_i fixed on the phalanx, by the vector ${}^f\mathbf{x}_i$. Note that, ${}^f\mathbf{x}_i \in \mathbb{R}^3$ is actually bounded to lie on the surface S_i (which is assumed regular) of the link, and therefore can be regarded as a mapping ${}^f\mathbf{x}_i \; : \; {}^f\alpha_i \in U_i \subset \mathbb{R}^2 \mapsto S_i \subset \mathbb{R}^3$. The pair $\left(U_i, \; {}^f\mathbf{x}_i({}^f\alpha_i)\right)$ is called a *chart* for (a portion of) the surface S_i, and the 2-vector ${}^f\alpha_i$ is referred to as the point coordinates on the i-th link. Orthogal coordinates can be chosen so that the associated metric tensor is diagonal. A normalized Gauss frame can be associated with each point on the surface chart that has the origin in the point and is fixed w.r.t to the body so that its ζ axis is aligned with the outward pointing normal, while the χ and ξ axes span the tangent space. The orientation of the Gauss frame centered in $\mathbf{x}_i$ w.r.t the C_i frame can be expressed by a rotation matrix ${}^f\mathbf{R}_i$. Similar considerations and definitions hold for the object surface.

Several types of contact models can be used to describe the interaction between the links and the object, among which the most common are the point-contact-with-friction model (or "hard-finger"), the "soft-finger" model, and the complete-constraint model (or "very-soft-finger"). In each case, the constraints consist in imposing that some components of the relative velocity between the Gauss frames that are associated with the contact point on each surface, are zero:

$$\mathbf{H}_i \; ({}^o\dot{\mathbf{c}}_i -{}^f \dot{\mathbf{c}}_i) = 0 \tag{1.2}$$

where $\mathbf{H}_i$ is a constant selection matrix. Being the two frames fixed on the object and the phalanx, respectively, their velocities can be expressed as a function of the velocities of the object and of the joints as

$$\begin{array}{rcl} {}^o\dot{\mathbf{c}}_i & = & \mathbf{G}_i^T({}^o\alpha_i, \mathbf{u}) \; \dot{\mathbf{u}}; \\ {}^f\dot{\mathbf{c}}_i & = & \mathbf{J}_i({}^f\alpha_i, \mathbf{q}_i) \; \dot{\mathbf{q}}. \end{array}$$

Similar relationships hold for each contact point, and a single equation can be built to represent all constraints by properly juxtaposing vectors and block matrices to obtain

$$\mathbf{H}\mathbf{G}^T\dot{\mathbf{u}} - \mathbf{H}\mathbf{J}\dot{\mathbf{q}} = \left[\begin{array}{cc} \mathbf{H}\mathbf{G}^T & -\mathbf{H}\mathbf{J} \end{array} \right] \left[\begin{array}{c} \dot{\mathbf{u}} \\ \dot{\mathbf{q}} \end{array} \right] = 0. \tag{1.3}$$

The matrix $\mathbf{G}$ is usually termed as the "grasp matrix", or "grip transform", while $\mathbf{J}$ is referred to as the hand Jacobian.

One of the goals of the kinematic analysis of manipulation systems is to explicit the relationships between joint positions and object positions. Eq. (1.3) can be used to this purpose (see e.g. [9]). Indeed, from (1.3), it is clear that the vector $[\dot{\mathbf{u}}, \; \dot{\mathbf{q}}]$ must belong to a certain linear space, and hence that there exist three vectors ν_1, ν_2, and ν_3 (whose dimensions vary with the problem at hand) such that every possible pair of object velocity $\dot{\mathbf{u}}$ and joint velocity $\dot{\mathbf{q}}$ that comply with the kinematic and contact constraints of the hand system can be written as

$$\begin{array}{rcccc} \dot{\mathbf{u}} & = & \mathbf{U}_o\nu_1 & + & \mathbf{U}_p\nu_2 \\ \dot{\mathbf{q}} & = & \mathbf{Q}_p\nu_2 & + & \mathbf{Q}_o\nu_3 \end{array} . \tag{1.4}$$

The columns of $\mathbf{U}_p$ and those of $\mathbf{Q}_p$ form a basis of the subspaces of compatible object and joint velocities, respectively. Any object motion described by the coordinate vector ν_2 in the image of $\mathbf{U}_p$ must correspond to a joint motion with the same coordinates in the basis $\mathbf{Q}_p$. The images of $\mathbf{Q}_o$ and $\mathbf{U}_o$ represent the subspaces of redundant joint velocities and under-actuated object velocities, respectively.

Note that the matrices appearing on the right hand side of (1.4) are functions of the position of the contact point on the surfaces. If the dependency between $\mathbf{u}$, $\mathbf{q}$ and ${}^o\alpha$, ${}^f\alpha$ is explicited via the kinematics of rolling (see e.g. [69, 58]), explicit expressions for the joint motions that are required to perform a desired object motion can be obtained in principle. Notice also that, besides the analytical difficulties, in practice we often have the case that the geometry of the object is poorly known, if at all. The availability of contact sensors that are able to provide information on the position of the contact points on the phalanges is therefore necessary to attempt closed loop control of fine manipulation. In particular, if joint angles and contact points are sensed, (1.4) can be used even without information on the geometry of surfaces to control the object motion about desired trajectories by using generalized resolved-rate control.

1.3 Grasp closure properties

In order to define what grasping robustness is, the notions of *form–closure* and *force–closure* of a grasp are instrumental. These properties, first introduced by [84], concern the capability of the grasp to completely or partially constrain the motions of the manipulated object, and to apply arbitrary contact forces on the object itself, without violating friction constraints at the contacts.

1.3.1 Form closure

Form–closure is the ability of a hand to prevent motions of the object, relying only on unilateral contact constraints. A mathematical definition of the problem can be stated as follows

Definition 1.1 *A configuration* $\mathbf{u}_0$ *of an object is form–closed by a hand in configuration* $\mathbf{q}$ *if* $\mathbf{u}_0$ *is an isolated solution of the contact inequalities (1.1), i.e., if for all* $\mathbf{u}$ *close to* $\mathbf{u}_0$*,* $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, \dot{\mathbf{u}}) \geq 0 \Rightarrow \mathbf{u} = \mathbf{u}_0$.

This purely geometric, rather general definition of form–closure can be specialized to allow easy–to–check tests. In particular, in many cases it will suffice to look at the first-order approximation of the contact inequalities, which under rather general circumstances can be written as

$$\mathbf{N}^T \mathbf{G}^T \dot{\mathbf{u}} \geq 0 \tag{1.5}$$

where $\mathbf{G}$ is the grasp matrix (evaluated at the current configuration) and $\mathbf{N}$ is a matrix stacking contact normal vectors in its diagonal. Hence we have the following cases

i) if there exists $\dot{\mathbf{u}}$ such that all components of $\mathbf{N}^T \mathbf{G}^T \dot{\mathbf{u}}$ are positive, the grasp is not form–closed;

ii) if for all $\dot{\mathbf{u}}$, $\mathbf{N}^T\mathbf{G}^T\dot{\mathbf{u}}$ has at least one strictly negative component, the grasp is form closed;

iii) if case i) does not apply, but there exists $\dot{\mathbf{u}}$ such that $\mathbf{N}^T\mathbf{G}^T\dot{\mathbf{u}}$ is nonnegative, the grasp may or may not be form–closed.

In cases i) and ii), second order terms are negligible, and form–closure can be decided by first order arguments, using for instance linear programming. Specifically, case ii) is termed "first–order form closure", and it corresponds to the most widely studied case in the literature. On the other hand, second or higher order effects must be taken into account in case iii).

First–order form–closure (which also has direct bearing to the design of mechanical fixtures and jigs for manufacturing parts) has been studied since the 19th century. Early results showed that at least four frictionless contacts are necessary for grasping an object in the plane, and seven in the 3D case. In [68] and [59], it was shown that four and seven contacts are necessary and sufficient for the form–closure grasp of any polyhedron in the 2D and 3D case, respectively. An active area of research is the *synthesis* of form–closure grasps, i.e., given the object geometry, where to place contacts so as to prevent object motions. Constructive procedures for placing contacts on given objects to achieve form–closure have attracted much attention in the literature, due also to the relevance to the fixturing problem (see e.g. the early work of [60], and more recently [33, 95, 11, 56, 55, 104]). There is also a form–closure *analysis* problem, i.e., given an object and a set of contact locations, to decide whether the object has any degree-of-freedom left, and which. Both qualitative (true/false) tests (see e.g. [51, 61, 68, 34]) and quantitative (quality index) tests ([48, 100, 65]) have been proposed for form–closure. The extension of the classical, first–order notion of form–closure to the so–called immobilization problem, where second–order effects due to the relative curvature of the surfaces in contact are taken into account, has been introduced rather recently to provide more detailed results (see e.g. [35, 86, 102]) in case iii) above.

1.3.2 Force closure

The analysis of form–closure is intrinsically geometric, and does not take into account the kinematics and characteristics of the end–effector. While there is a wide consensus in the literature on the definition of form–closure, the concept of force–closure is somewhat less clearcut and universally accepted. The intuitive meaning of force–closure implies that motions of the grasped object are completely (or partially) restrained despite whatever external disturbance, by virtue of suitably large contact forces that the constraining device (the end-effector) is actually capable to exert on the object.

The force and moment balance equations for an object subject to an external force $\mathbf{f}$ and moment $\mathbf{m}$, while grasped by a robotic mechanism by means of n contact forces $\mathbf{p}_i$ applied at contact points $\mathbf{c}_i$, is written as

$$\mathbf{w} = \mathbf{G}\mathbf{p}, \tag{1.6}$$

where $\mathbf{w} = (\mathbf{f}^T, \mathbf{m}^T)^T$ is the external wrench, and $\mathbf{p} = (\mathbf{p}_1^T, \ldots, \mathbf{p}_n^T)^T$. The relationship between contact forces and the torques at the m joints of the robotic hand can be written using the hand jacobian as

$$\tau = \mathbf{J}^T \mathbf{p},$$

A general solution of (1.6) can be written in the hypothesis that $\mathbf{w}$ is *resistible* (i.e., that $\mathsf{rank}\ \mathbf{G} = \mathsf{rank}\ [\mathbf{G}\ \mathbf{w}]$) as

$$\mathbf{p} = \mathbf{G}^R \mathbf{w} + \mathbf{A}\mathbf{x}, \tag{1.7}$$

i.e., the sum of a particular solution of (1.6) ($\mathbf{G}^R$ is a right-inverse of $\mathbf{G}$), and a homogeneous solution. $\mathbf{A}$ is a matrix whose column form a basis of the nullspace of $\mathbf{G}$. The coefficient vector $\mathbf{x} \in \mathbb{R}^{h_0}$ parametrizes the homogeneous solution. Internal contact forces $\mathbf{p}_h = \mathbf{A}\mathbf{x}$ have no direct effect on the external wrench $\mathbf{w}$, but play an important role in the robustness of the equilibrium with respect to slippage induced by external disturbances, by allowing to "squeeze" the object in the grasp. It should be noted that, in general, for grasping mechanisms with few degrees of freedom , it may not be possible to apply arbitrary internal forces (see below section 1.7).

In force–closure analysis one generally has to deal with frictional contacts. In different models of contact, such as the contact-point-with-friction, soft-finger, or very-soft-finger, friction forces and torques will be subject to limitations due to Coulomb's law of friction or to its generalizations (see e.g. [30, 38]). We consider here contacts of the first type (generalization poses no difficulties), for which Coulomb's inequality holds,

$$\sigma_{i,f}(\mathbf{p}_i) \ = \ \alpha_i \|\mathbf{p}_i\| - \mathbf{p}_i^T \mathbf{n}_i < 0, \tag{1.8}$$

representing a cone in the space of contact forces $\mathbf{p}_i$. Substituting (1.7) in (1.8), an expression of friction constraints in terms of external wrenches and internal forces $\sigma_{i,f}(\mathbf{w}, \mathbf{x}) < 0$ is obtained. In these terms, we can state the following

Definition 1.2 *A grasp is defined* Force–Closure *if, for any external wrench* $\mathbf{w}$ *acting on the object, there exists a vector* $\mathbf{x}$ *such that all friction constraints are fulfilled.*

The analysis of force–closure has been considered among others by [74, 28, 18, 73], while literature on the synthesis of force–closure grasps include [74, 79, 80, 81, 6].

According to the previous discussion on force–closure, a crucial problem in robot manipulation is the choice of grasping forces so as to avoid (or minimize the risk of) slippage. The problem of choosing joint torques so as to realize the manipulating forces required by the task, while imposing internal forces that guarantee slippage avoidance, is often referred to as the *force distribution* problem. Further constraints on the choice of contact forces come from limitations in the object strength, or in the joint actuators torques. Accordingly, an "optimal" set of internal forces can be defined as the one that is further away from violating all such constraints. The force distribution problem is common with other robotic areas, as e.g. legged locomotion, cooperating and/or constrained manipulation, and has attracted much attention in the past few years (see e.g. [76, 47, 42, 54, 73, 105, 43, 78, 13]).

An important property of the nonlinear constrained optimization problem to which grasp force distribution amounts is convexity. This property, used first in [6], enables efficient solutions to an otherwise very complex problem: [6] proposed numeric integration of an ODE as an iterative solution to the problem; [15] noticed that nonlinear friction constraints can be rewritten as positive–definiteness constraints on suitable matrices, and used projected gradient flow methods to optimize; [52] further exploited the matrix formulation of [15] to transform the problem in the format of a standard linear matrix inequality (LMI) problem, for which off–the–shelf, effective software exists.

1.4 Dynamics

The ability to predict the dynamic behavior of a grasp with a given model including the control algorithms, is critical to the design of the grasp. In multifingered grippers, as in legged locomotion systems, multi-arm systems, and other constrained robot systems, several limbs are used to constrain and manipulate an object [50, 54, 67]. The dynamic analysis and the simulation (the prediction of motion given the external forces and moments on the system) of such systems is central to the design of such systems and the development of control algorithms [106, 90].

A hand-object system is a constrained mechanical system, whose dynamical description can be derived using Euler–Lagrange's equations along with constraint equations. The disjoint dynamics of the hand and of the object are written as

$$\mathbf{M}_h(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{Q}_h(\mathbf{q}, \dot{\mathbf{q}}) = \tau;$$

$$\mathbf{M}_o(\mathbf{u})\ddot{\mathbf{u}} + \mathbf{Q}_o(\mathbf{u}, \dot{\mathbf{u}}) = \mathbf{w},$$

where $\mathbf{M}_h(\cdot)$ and $\mathbf{M}_o(\cdot)$ are symmetric positive definite composite inertia matrices, and $\mathbf{Q}_h(\cdot,\cdot)$ and $\mathbf{Q}_o(\cdot,\cdot)$ are terms including velocity-dependent and gravity forces of the hand and of the object, respectively. Hand and object dynamics are linked through the n rigid–body contact constraints (1.1).

As we have seen before, when there are contacts between nominally rigid bodies, contact constraints are unilateral. Featherstone [27], Lotstedt [57] and Mason and Wang [62] pointed out some of the inconsistencies which arise when rigid body models are used with Coulomb's empirical law of friction in unilateral systems. For example, if we consider the simulation of a rod sliding along a rough ground in a plane with a single contact, there are configurations in which no solutions (that are consistent with the constraints) exist, and others in which the solution is not unique. Wang, Kumar, and Abel [107, 106] performed a dynamic analysis of the peg-in-the-hole insertion problem and showed that there was a range of parameters during two-point-contact for which there were either no solutions or two solutions for the accelerations. Quasi-static analysis is also known to exhibit such inconsistencies [36].

The inconsistencies and ambiguities in the dynamic analysis of frictional contacts have been attributed to the approximate nature of Coulomb's model and to the incorrect assumption of rigidity. Recently, there has been some attention in the robotics community on overcoming these shortcomings by using rigid body models to predict the gross motion while using compliant contact models to predict the contact forces and the local deformations [49].

One of the main difficulties that is present in multifingered grasps, and a feature that is particularly true of such grasps as power grasps and enveloping grasps, is that the number of independent contact forces is much larger than the number of actuators. Thus, from a controllability standpoint, not all the contact forces are controllable (see below section 1.7).

The analysis of statically indeterminate grasps or grasps in which there is no unique solution to the inital value problem is simply not possible unless one explicitly models the compliance at the contacts [20, 36, 74, 49]. Of course such contact models tend to be more complex and the parameters are more difficult to identify (see below section 1.6). Further, it is harder to simulate systems in which the time scale for the dynamics of contact interactions is significantly different from the time scale of rigid body dynamics [63, 97]. Thus, although efficient, approximate algorithms for "impulsive dynamic simulation" that incorporate approximate impact models for collisions are available [66], it is very difficult to write accurate simulators for dexterous and fine manipulation where the contact forces may be finite and the results may be sensitive to the parameters in the contact model.

1.5 Stability

A further important property of grasps is *stability.* The term is used in the literature with at least two meanings. One refers to Lyapunov theory, and dictates that a grasp is (asymptotically) stable if its dynamics are such that, when the object is displaced from its reference position, it will stay close (and ultimately come back), to such position. A second definition is Lagrange's, whereby a grasp in which all forces are conservative, is stable if it corresponds to a strict local minimum of the potential energy. The second usage is prevalent in studies on grasp stability.

It is important to note that force closure does *not* guarantee stability. Any definition of stability must regard the grasp as a dynamic system and describe the properties of the dynamic system when it is perturbed from an equilibrium configuration. The role of compliance and dynamics in grasping has been investigated by many authors, beginning with Hanafusa and Asada [32] and Salisbury[61]. Cutkosky and Kao [21] discussed how to compute the aggregated compliance matrix of a hand–object system, including finger flexibility effects. Relations of compliant and rolling contacts with the stability of the grasp have been considered, at increasing levels of generality and detail, by

[22, 70, 100, 36, 99, 29]. If Lagrange's stability criterion applies to an equilibrium grasp for a conservative system, Lyapunov stability follows. It should be noted however that Lagrange's analysis is limited under some regards. In mechanics, the seemingly intuitive statement that, if an equilibrium point is not a minimum for the potential function, then it is unstable, does not have a proof for systems with more than 2 d.o.f. [3]. Perhaps more importantly, from an application viewpoint, is the fact that no provision is made in Lagrange analysis for non–conservative forces (except for Rayleigh–type dissipative terms). Nonconservative forces may arise in grasping systems because of nonidealities in the mechanical components, and of the control laws used for actuating the hand joints. The inclusion of the effects of control on the stability of grasp, which are apparently of major moment, is as of today a mostly open research problem. Lyapunov stability, and other structural properties (controllability, observability, stabilizability) of general grasping systems in their linear approximation have been investigated by [12, 82, 2]. Stable control of manipulation and grasping systems has been considered among others by [72, 85, 92, 87]. Particularly important is work done towards controlling grasping systems in the (practically ubiquitous) presence of uncertainties ([17, 24]).

A figure measuring stability (useful e.g. to compare different possible grasps) may be considered ([36]) as the real part of the dominant eigenvalue of the linearized grasp model (large values of this measure indicate that small perturbations are damped away quickly). An even more useful figure, in many applications, would be related to the size of the basin of attraction of the equilibrium, indicating how large a perturbation can be without causing instability: however, effective algorithms to evaluate such measure are not available at present.

1.6 Contact compliance.

The importance of modeling the finger-object contact and the role of compliance in grasping has been stressed by many researchers [4, 20, 94]. However, it is particularly difficult to model the relationship between small object/finger displacements and changes in contact forces arising from these displacements.

Such contact problems have been studied extensively in the solid mechanics community in the context of rail-wheel interaction [45] and analysis of ball and roller bearings [44]. There are difficulties even in establishing the uniqueness and existence of solutions of elastic bodies in static contact [25], and tractable analytical models are, in general, very difficult to come by. Hertz's model [44]can be used to predict the pressure distribution across each contact patch when the contacts are frictionless and non-conformal. Hertzian contact theory is probably the most widely used analytical contact model, and variations of this are used in [36, 86].

Because friction is central to robotic grasp, the Hertzian contact model has proved to be inadequate in many cases. Sinha and Abel [94] proposed an elastic contact stress model for finger-object contacts in multifingered grasping and a variational approach for quasi-static analysis. Wang, Kumar, and Abel [107] proposed a similar approach for dynamic analysis. They developed a mathematical programming approach for frictional, elastic contacts as well as viscoelastic contacts in which the inertial forces due to the deformations at the contacts are neglected. While such distributed parameter models yield accurate results, the solutions require computation-intensive numerical methods. A possible simplification is provided by the Winkler elastic foundation model [44], and the lumped parameter visco-elastic models used in [30, 49, 97] provide the simplest model for simulation and analysis.

One of the very hard problems is getting an accurate and tractable model of contact compliance, particularly in the tangential direction. This is recognized to be a difficult problem in the mechanics literature as well [44]. In addition to this, a tractable and accurate model of friction, one that accurately predicts slip and one that lends itself to stability analysis, is currently not available. Both these fundamental problem areas are crucial to robotic grasping and contact analysis.

For the purposes of analysis of grasp, it is generally assumed in the literature that all contacts are point contacts and idealizations such as a line or surface contact can be approximated by two or more point contacts. Each point contact can be modeled as either a frictionless point contact, a frictional point contact, or a soft contact [88]. A frictionless contact is defined as a contact in which the finger (or effector/fixture) can only exert a force along the common normal at the point of contact. A frictional contact (sometimes referred to as a point contact with friction) is defined as a contact that can transmit both a normal force and a tangential force, while a soft contact also allows the finger to exert a pure torsional moment about the common normal at the point of contact.

1.7 Grasping and the kinematics of the hand

It is interesting that much of the literature in grasping actually ignores the kinematics of the fingers or the articulations that are involved in contacting the object. While Reuleaux's problem of form closure justifiably focused on the geometry of the object and the arrangement of contacts, it is difficult to analyze a grasp without modeling the dynamics, or at least the kinematics, of the fingers and the interaction of the fingers with the object.

Trinkle *et. al.* explore the kinematics of enveloping grasps [101] using the restrictive but conservative assumption of frictionless contacts. The kinematics of fingers with two or three point contacts with fingertips and palms have been studied by [77, 26]. While the analysis of form–closure is intrinsically geometric, force–closure is tightly linked to the kinematics and characteristics of the end–effector. In fact, it is possible that a geometric analysis of a grasp may predict force–closure, but a careful analysis of the kinematics may reveal that this is not the case [35]. Definitions of force–closure that take into account the kinematics of the gripping device were proposed in [6], along with an exact algorithm for testing such property. Yoshikawa proposes a new set of definitions for closure properties, including what he calls active and passive closures, to explicitly model the properties of the grasping mechanism [109]. Unfortunately, much of this, and other related work [39] is based on instantaneous kinematics.

Modeling of the fingers is particularly important when end–effectors that have fewer degrees–of–freedom than necessary to impart arbitrary motions/forces at all contacts. Such *kinematically defective* grasps are common in simple industrial grippers. If the hand Jacobian matrix is not full rank, it is not possible to command an arbitrary set of grasp forces [5]. This is usually the case in all power grasps. The modeling of the kinematics and manipulability of whole–hand manipulation in such systems is discussed in [83]. Intuitively, the more a grasp is defective, the more robust it is in restraining an object with respect to external disturbances and the lower is sensitivity to positioning errors, but also the lower is manipulability. However, a case-by-case analysis is necessary for optimal power grasps [6].

Many open problems remain to be solved in order to be able to design robot hands to effectively exploit defectivity to increase grasp robustness and reduce hardware complexity. Among these, perhaps the most important is the need for a reliable estimate of contact compliance, arising with statically indeterminate grasps. This will then allow the calculation of contact forces, and the development of models that relate joint displacements and torques to contact forces.

1.8 Measures of grasp performance

Recent work in the literature has tried to develop quality measures for grasps. One such measure can be derived from the conditioning of the grasp matrix and is directly connected with the closure properties of the grasp [54]. In a similar fashion, other structural properties can be derived from the characteristic matrices, for example, controllability and observability [83].

When an object is restrained or grasped with multiple effectors, there are two, often conflicting, measures of grasp performance. First, if the fixtures can be accurately positioned, the system's ability to reject wrench disturbances is a measure of grasp stability. The grasp stiffness matrix, or a frame invariant measure of the minimum grasp stiffness [14], provides one choice for a performance metric. This assumption of being able to accurately position the end-effector is extensively used in the fixturing and grasping literature. However, when there are errors in positioning and orienting the end-effectors, it is important to choose a grasp so that the system performance is insensitive to these positioning errors. Thus, it also makes sense to minimize the dependence of grasp forces on such positioning errors.

Howard and Kumar [36] develop the theory needed to combine the stiffness matrices at each contact to calculate a grasp stiffness matrix. While the signs of the eigenvalues allow a test of grasp stability, the eigenvalues themselves are not invariant with respect to changes in reference frames [35]. Bruyninckx *et al.* [14] develops a frame invariant measure of stability that is based on the grasp stiffness matrix and a metric on the Euclidean group. Lin develops a frame-invariant quality measure that essentially minimizes the "object deflection" when the grasped object is subject to force disturbances [55]. The basic idea here is to scale the eigenvalues measuring the rotational stiffness by a characteristic distance to an edge of the object. Thus it is possible to develop a scaled stiffness matrix and the smallest eigenvalue of the scaled matrix characterizes the system.

The focus in the above work is to quantify the ability of a fixture to reject disturbances due to external forces on the workpiece [23]. This is clearly a measure of performance that is relevant. However, the robustness of a grasp to errors in positioning the effectors has not been addressed in this literature. Sugar and Kumar develop a second measure of performance that characterizes this robustness and discuss an approach to optimizing fixtures based on both measures [98]. In this connection, the control of grasping and the effects of of uncertainties are particularly important.

Unfortunately most of these measures are based on the assumptions of small perturbations: displacements, forces and errors. There is no question that more global measures would be more useful. For example, in stability analysis, a figure relating to the size of the basin of attraction of the equilibrium, indicating how large a perturbation can be without causing instability would be desirable. However, the nonsmooth nature of grasp dynamics (because of the unilateral constraints on displacements and forces) has made a thorough analysis very difficult.

1.9 Concluding Remarks

This chapter presented a survey of work in robotic grasping and manipulation over the last twenty years. It is impossible to do justice to all the work in this area, particularly because of the breadth of the field and its close connection to dexterous manipulation, fixturing, and haptics. We chose to focus on issues that are central to the mechanics of grasping and the finger–object contact interactions. In addition, the review mainly addressed research that has established the theoretical framework for grasp analysis, simulation and synthesis. Because of the limitations on space, we have not given the algorithmic aspects, and the applications the attention that they deserve.

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