“The stress of shear"

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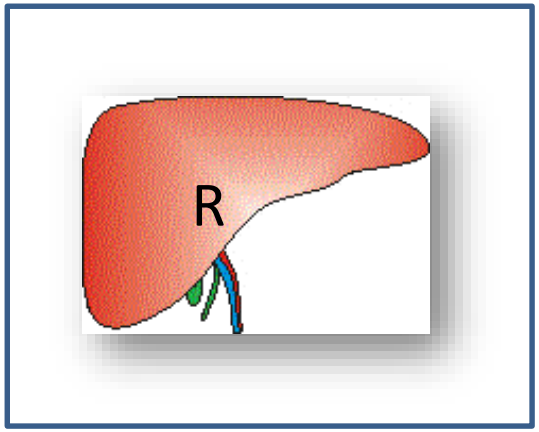
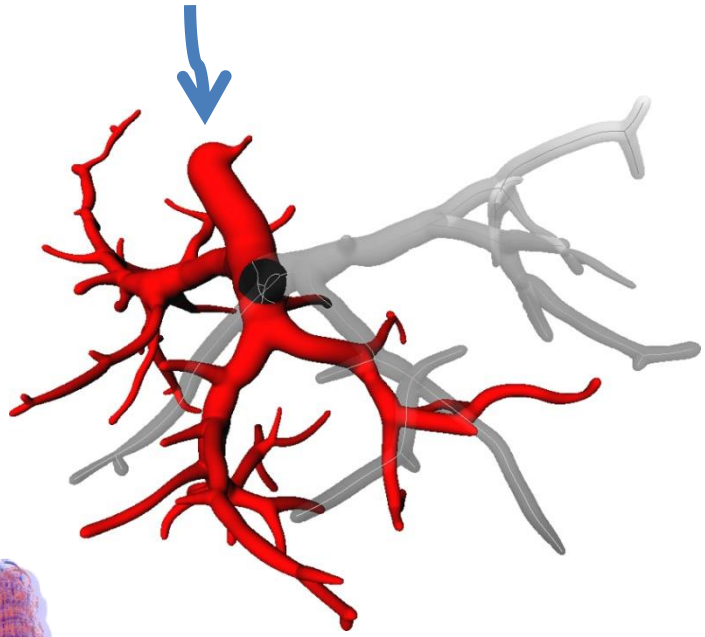
Blood flow in, Q_{in}

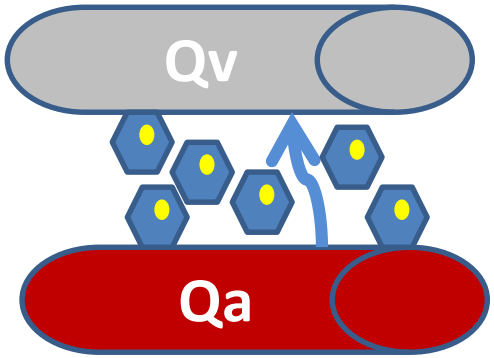
C_{in}



Blood flow out, Q_{out}

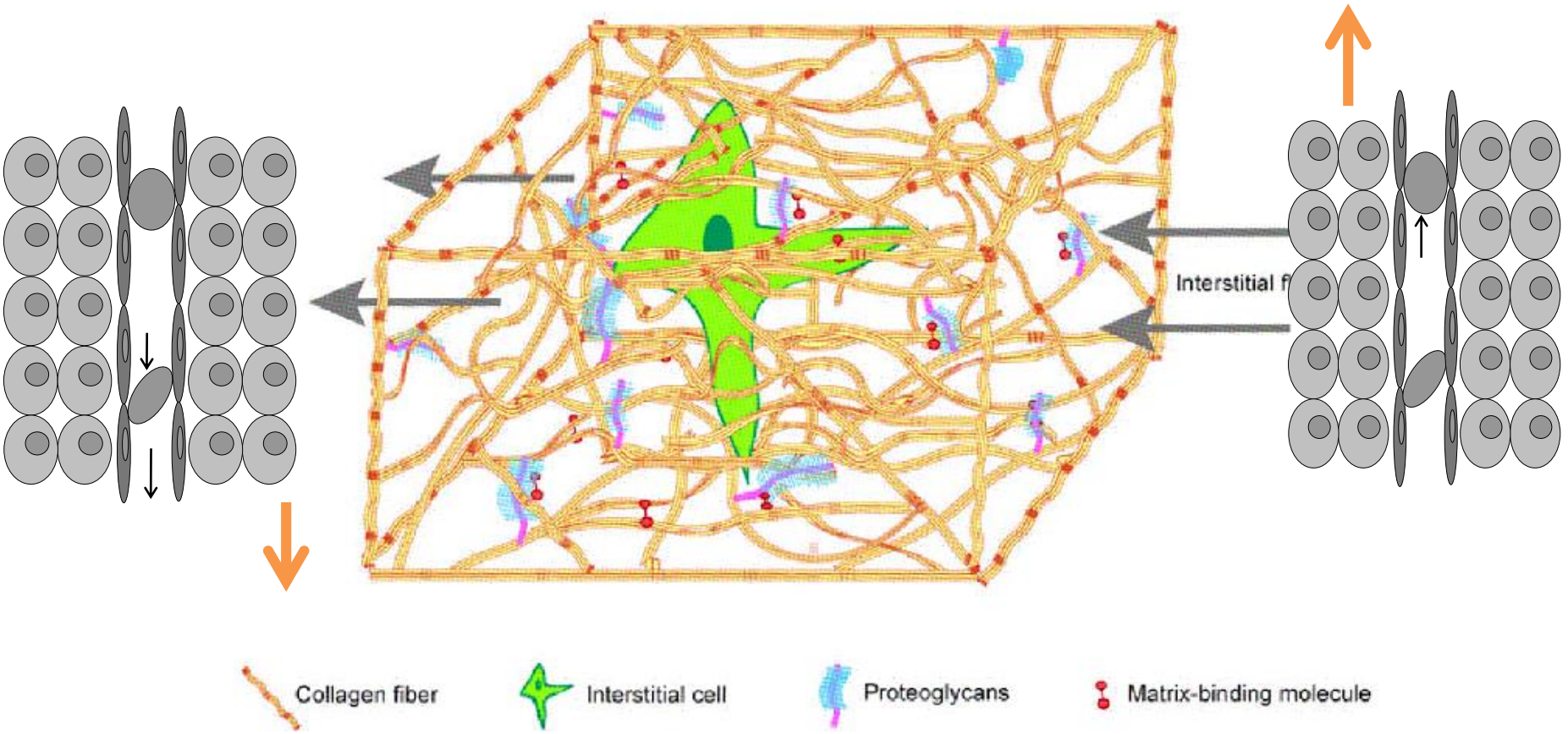
C_{out}





Interstitial flow driven by concentration gradients

INTERSTITIAL FLOW

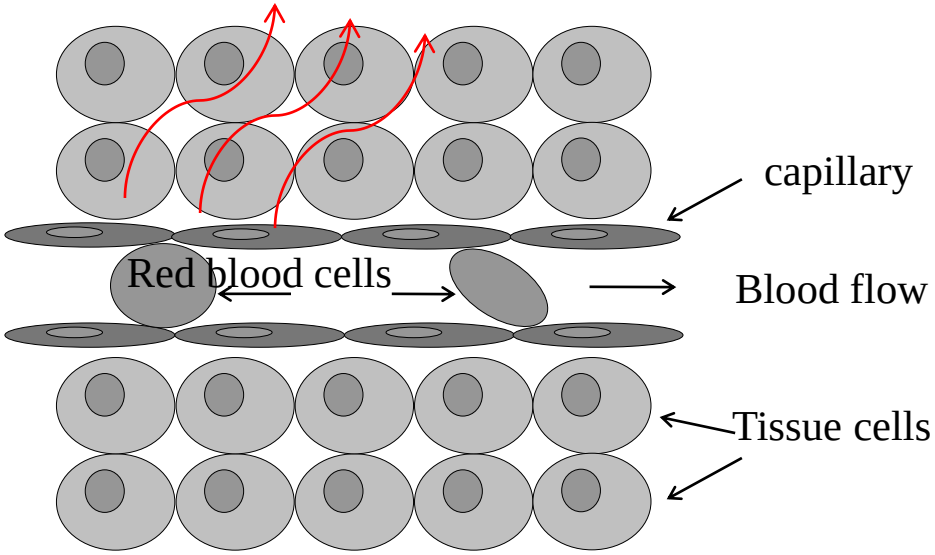


1) interstitial flow is due to a concentration gradient 2) all tissues are permeated by interstitial flow 3) the flow is through a microporous medium

Swartz & Fleury, ARBE
Vol. 9: 229-256.2007

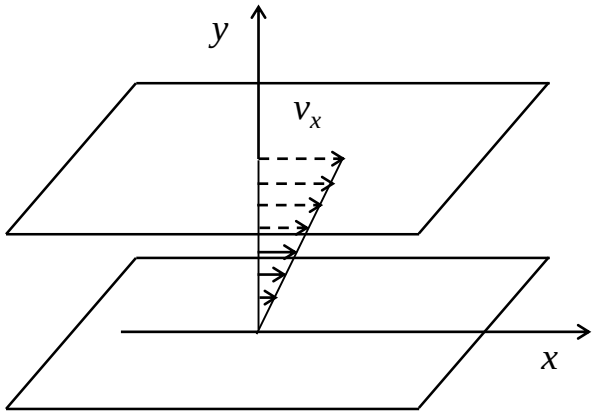
FLOW and SHEAR

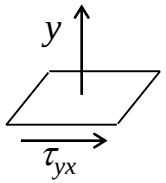
Only epithelial cells (skin, blood vessels, intestine) and the non adherent cells of the immune system and blood can support direct fluid flow.



The motion of fluid across a mobile or semi mobile surface gives rise to **shear stress**

Shear stress





$$\tau_{yx} = -\mu \frac{dv_x}{dy}$$

The shear stress on a monolayer of cells in a flat chamber with flow Q is

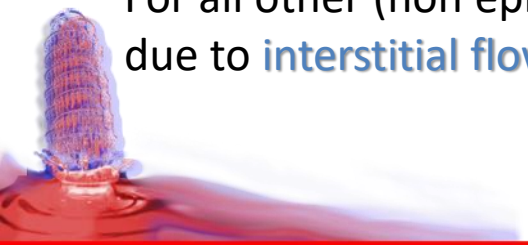
$$\tau_{yx} = -\frac{6Q\mu}{wh^2}$$

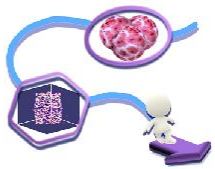
Optimal shear stress in bioreactors

Cell	Shear	Flow rate	Ref
Human trabecular bone, 3D	5.10^{-5} Pa	0.01 mL/min	Porter. Journal of Biomechanics, 38, 543, 2005
Human osteosarcoma cells, 3D	0-0.021 Pa	Max. 25 mL/min	Laganà.Biomedical Microdevices, **14(1), 225, 2012**
hBMSC, 3D	0.015 Pa	3 mL/min	Li. Tissue Eng. A, **15, 2773,2009**
HepG2, 2D	0.14 Pa	0.0025 mL/min	Tanaka et al, Meas. Sci. Technol. **17 ,3167–3170, 2006**
Human hepatocytes, 2D+ gel	5.10^{-5}Pa	0.25 mL/min	Vinci et al. Biotech J., 6(5):554, 2011
Rat hepatocytes, 2D+ fibroblasts	0.014 Pa	0.06 mL/min	Tilles et al, Biotech & Boeng. 73 (5),379 ,2001

Wall shear stress in blood vessels: 1-0.01 N/m^2

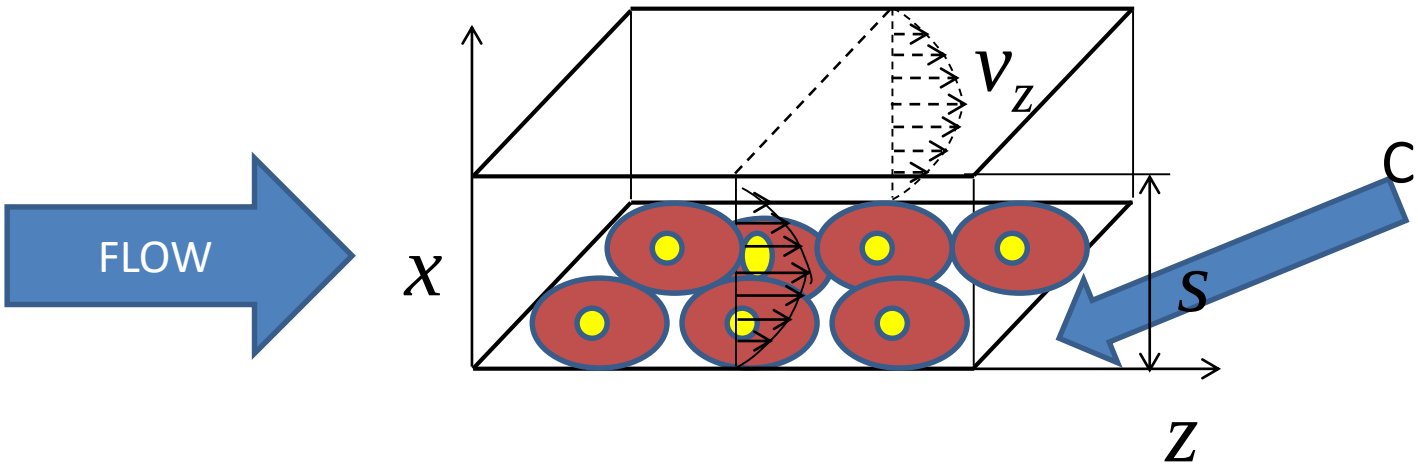
For all other (non epithelial) tissues shear is much less (0.01-0.00001 N/m^2), and is due to interstitial flow (few microL/min).





Adding flow

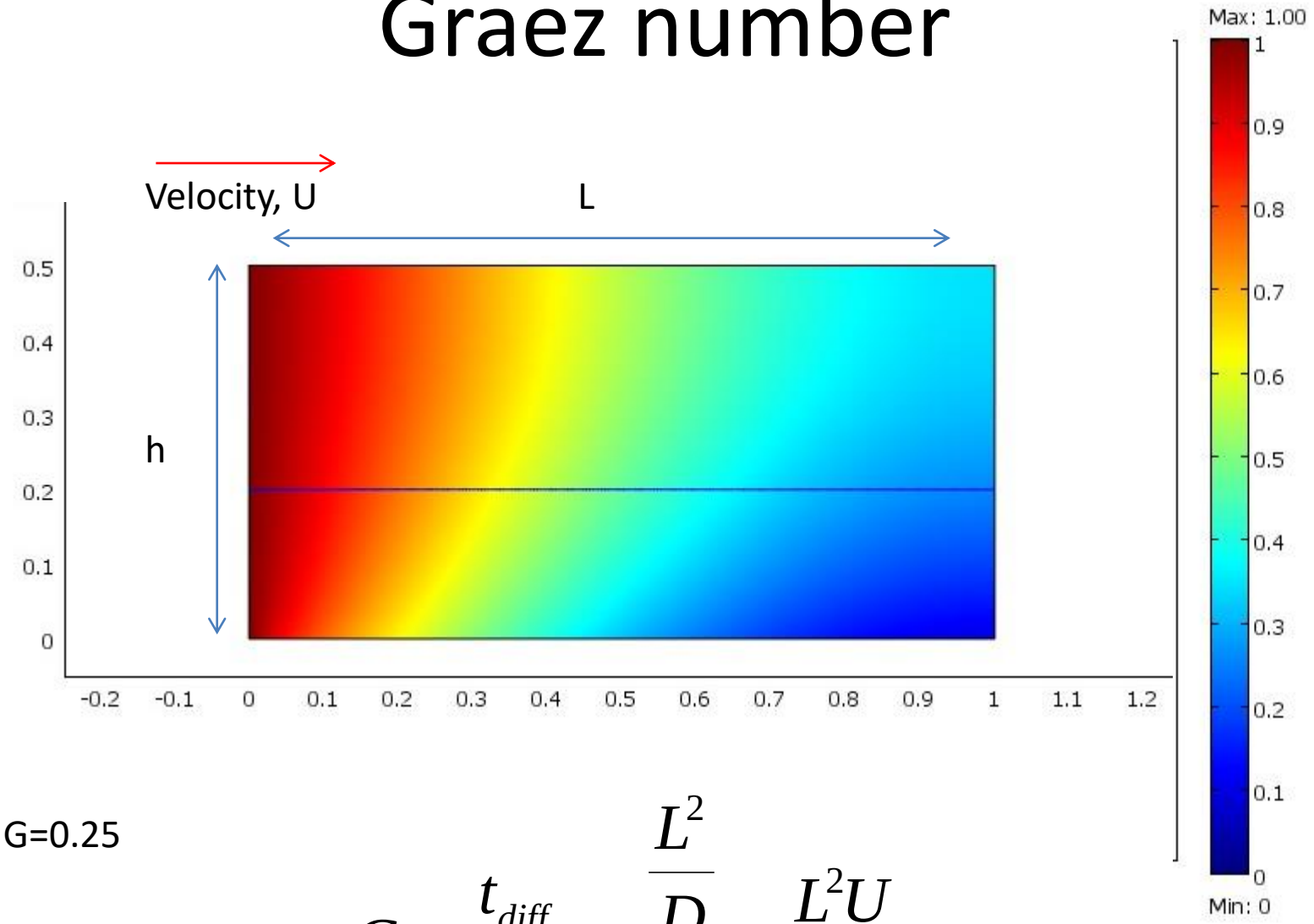
$$\frac{dc}{dt} = D\nabla^2 c - R - v.\nabla c$$



For a monolayer $$\frac{dc}{dt} = D\frac{dc^2}{dx^2} - v_z\frac{dc}{dz}$$

For volumetric consumption $$\frac{dc}{dt} = D\frac{dc^2}{dx^2} - v_z\frac{dc}{dz} - R_{vol}$$

Graez number

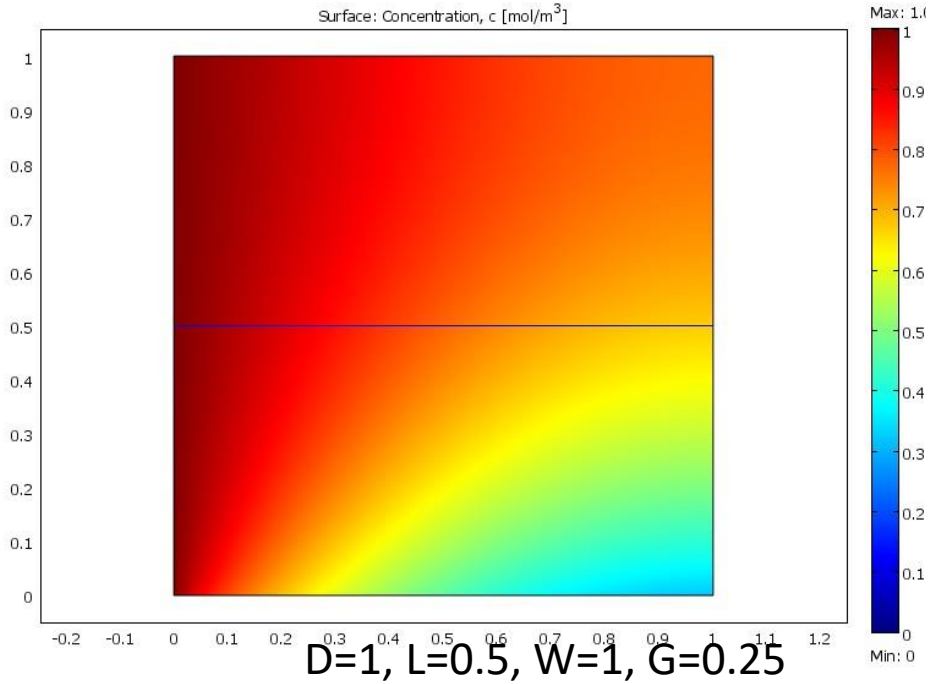


G=0.25

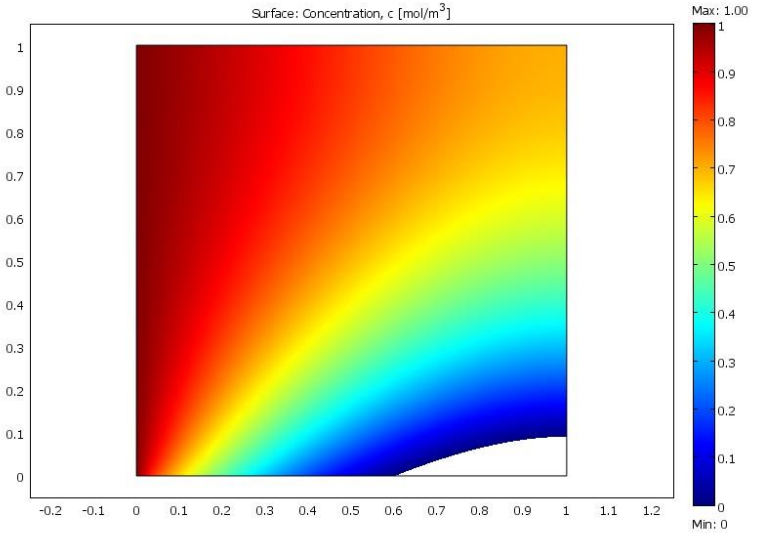
$$G = \frac{t_{diff}}{t_{conv}} = \frac{\frac{L^2}{D}}{\frac{h}{U}} = \frac{L^2 U}{Dh}$$

HOW CONCENTRATION PROFILES CHANGE WITH GRAEZ NUMBER

D=1, L=1, W=1, G=1



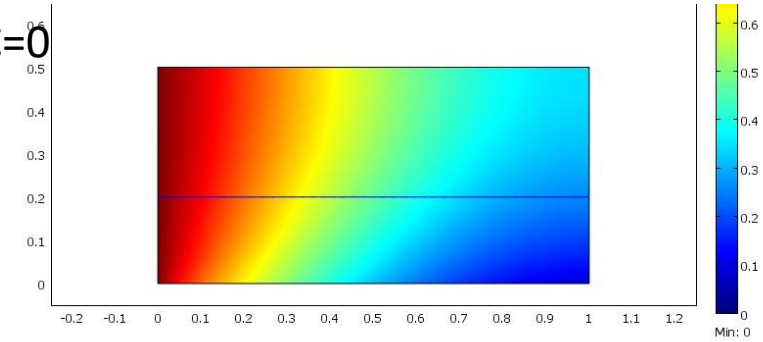
D=2, L=1, W=1, G=0.5



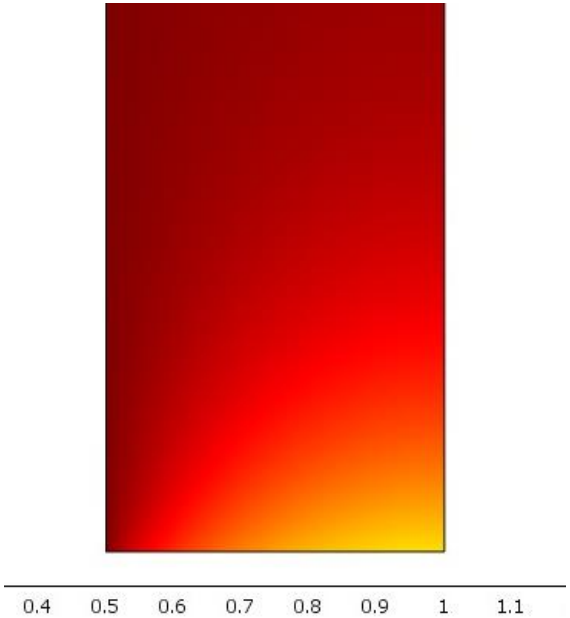
D=1, L=0.5, W=1, G=0.25

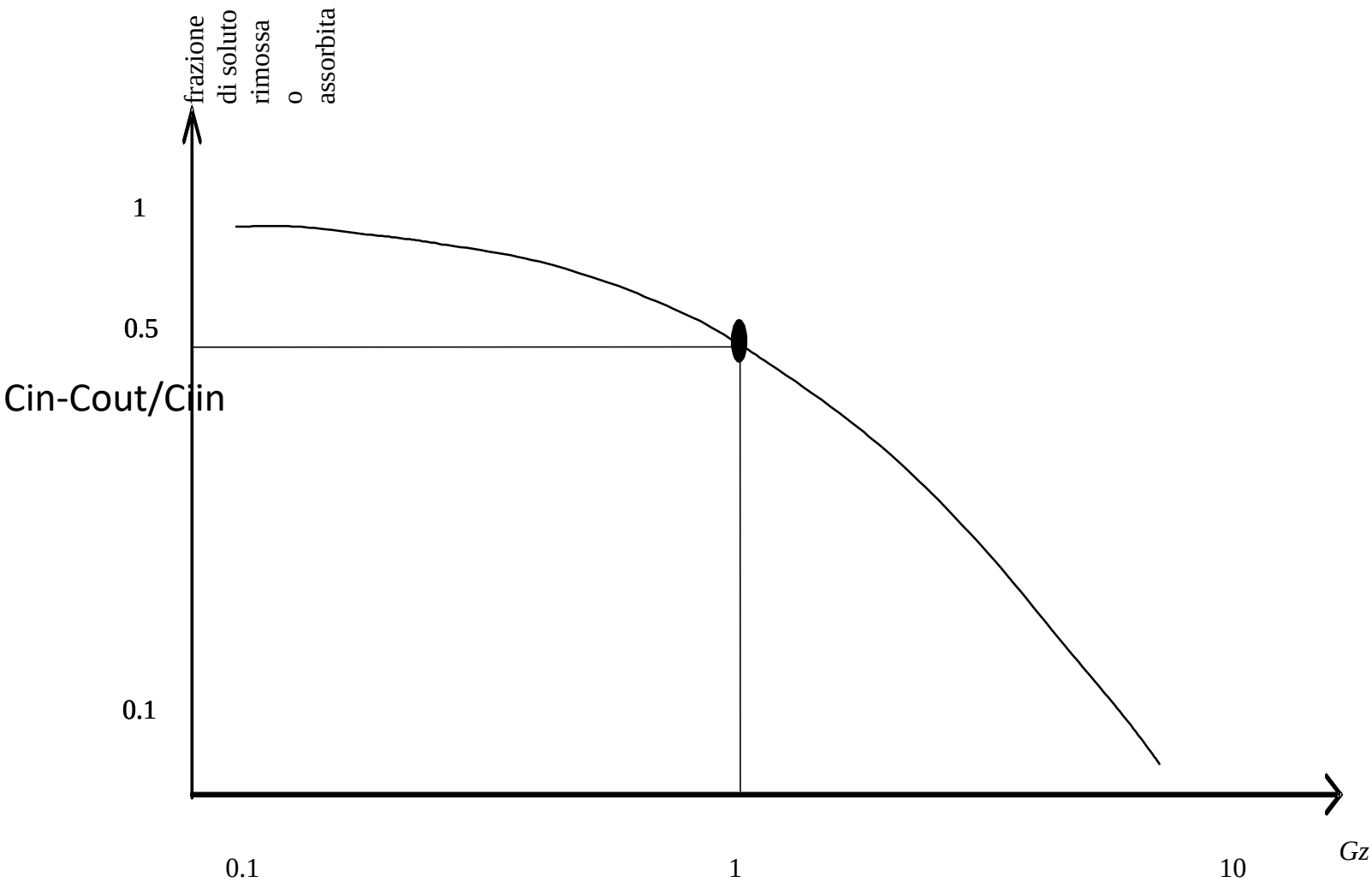
RED, C=1

BLUE C=0



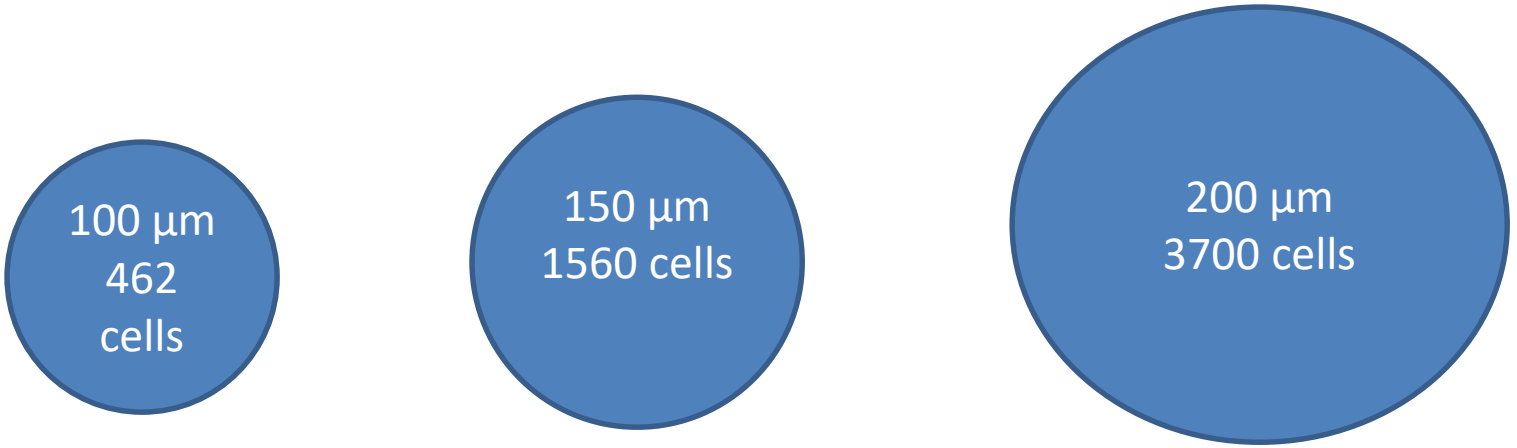
D=1, L=1, W=0.5, G=2




frazione di soluto rimossa o assorbita
1
0.5
Cin-Cout/Ciin
0.1
0.1
1
10
Gz

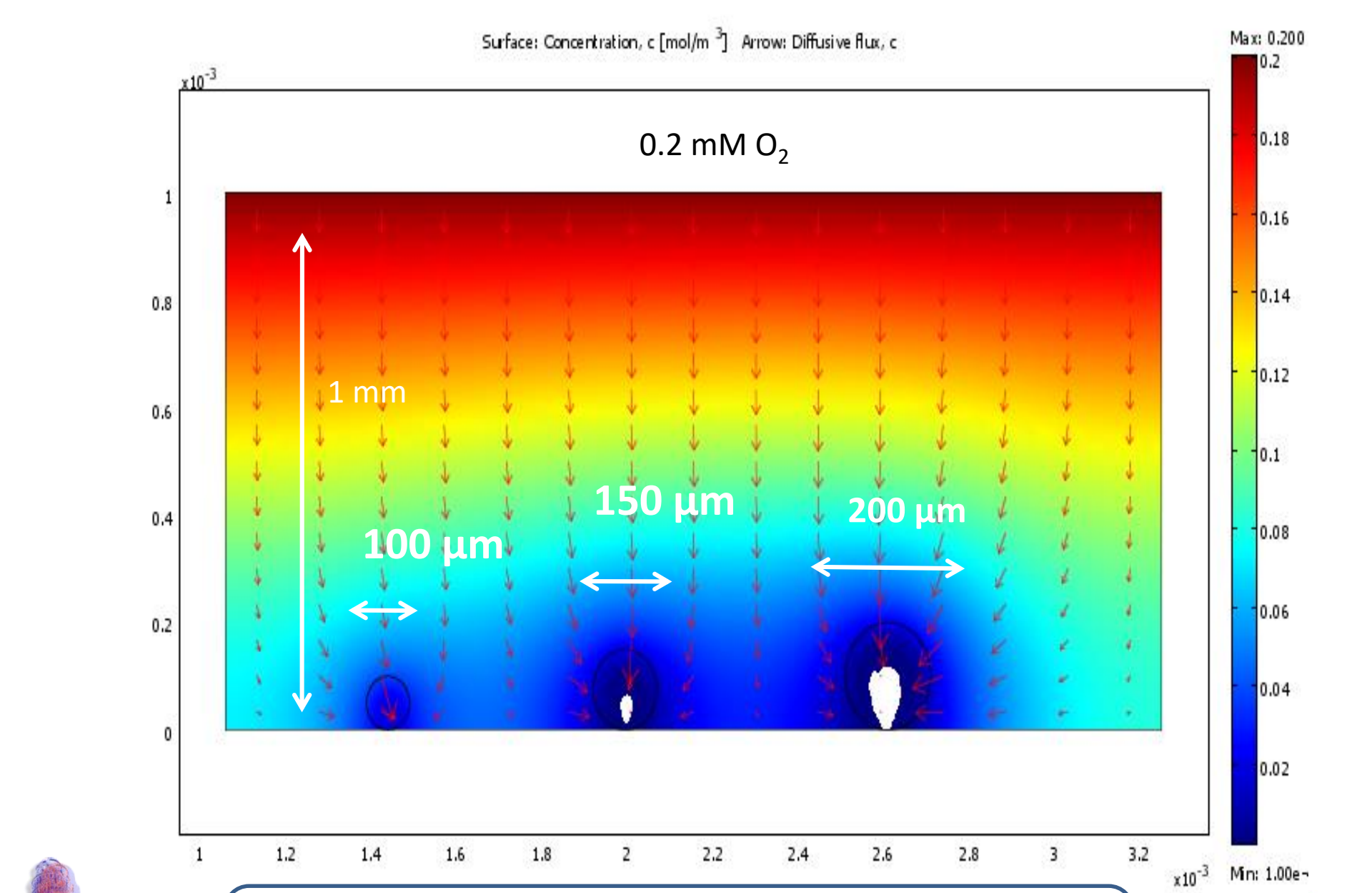
Stop here

Example 1: Oxygen diffusion in gel encapsulated islets



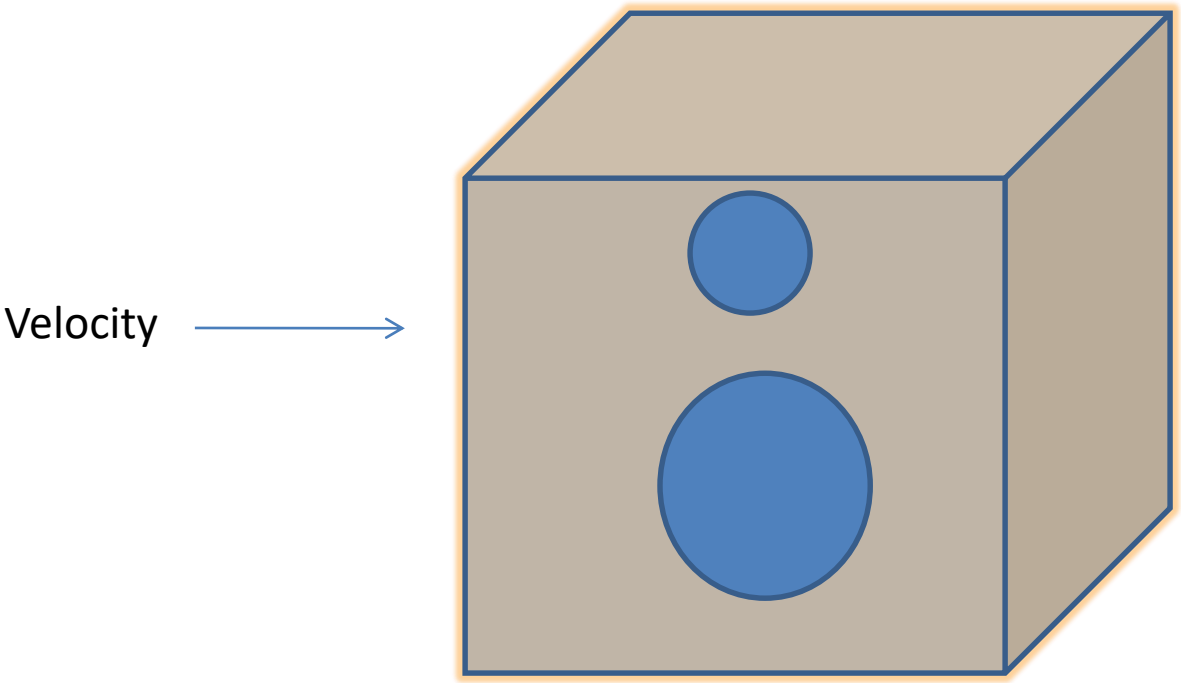
Vmax	Km	C crit	Co	D in water	D in sphere
0.034 mM.s^{-1}	**1.10^{-3} mM (0.7 mmHg)**	**1.10^{-4} mM (0.07 mmHg)**	**0.2 mM**	**3.10^{-9} m^2.s^{-1}**	**2.10^{-9} m^2.s^{-1}**
Medium height	**δ (Heavyside)**	**Cell density**			
1 mm	**flc1hs(c-0.02,0.0001)**	**8.8.10^{14} cells·m^{-3}**			

$$R_{vol} = \frac{V_{max} c}{K_m + c} . \delta$$



Simple problem solved with mass transfer equations in Comsol Multiphysics

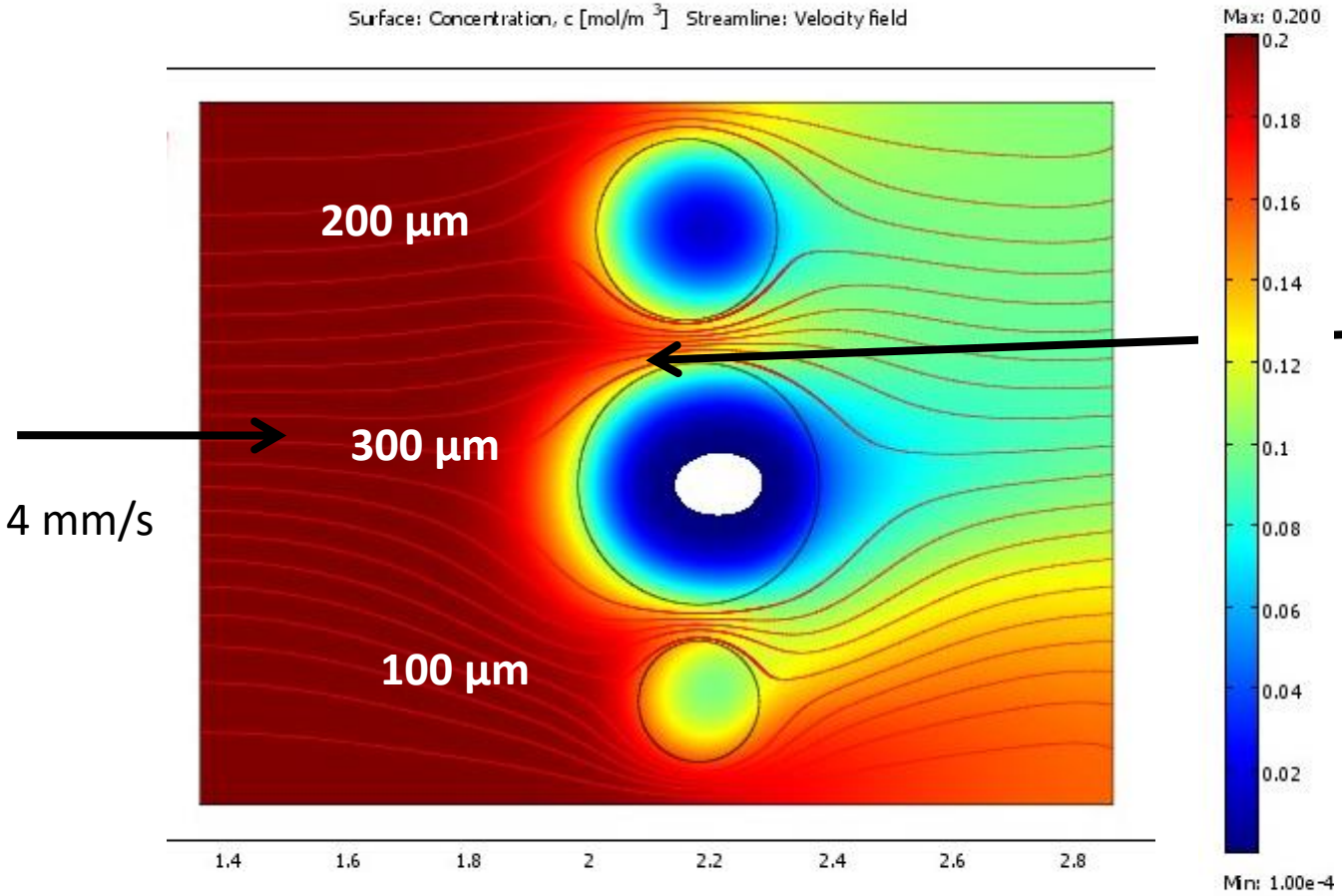
Example 2: Oxygen diffusion in perfused gel encapsulated islets in a bioreactor



The islets are encapsulated in a non porous gel
Nutrients will only get to the cells by diffusion through the gel

Solved by coupling the Navier-Stokes equations for the fluidic domain with convection and diffusion

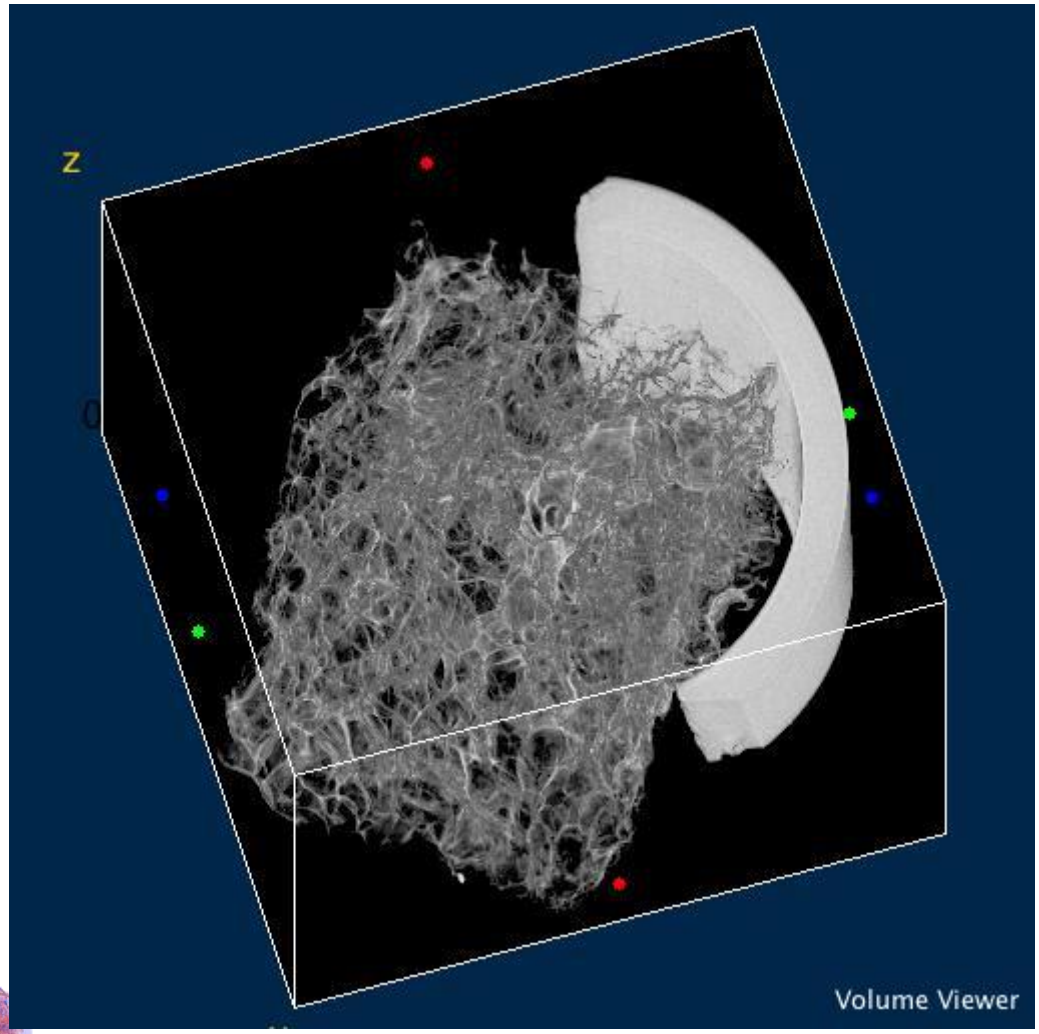
Islets in a bioreactor perfusion chamber, flow velocity of 4 mm/s.

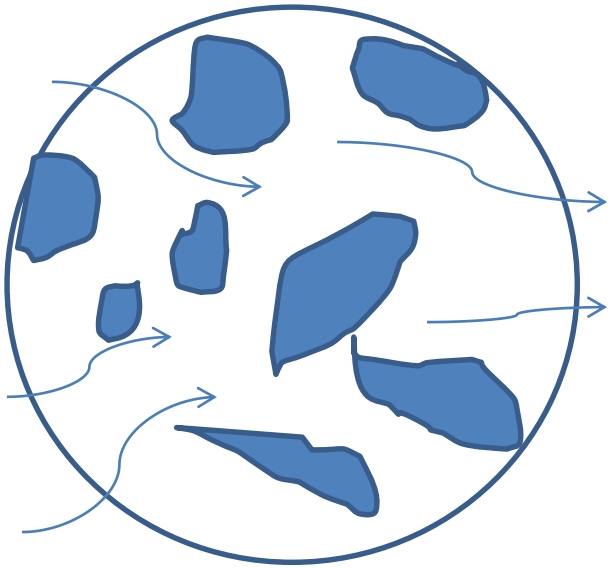


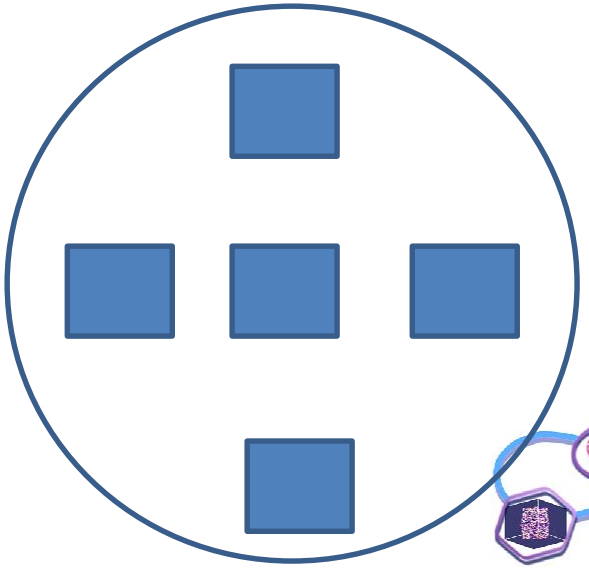
Maximum shear stress=0.085 Pa (on the high side)

The size limit is between 200 and 300 microns
- larger constructs have to be porous

Flow through pores



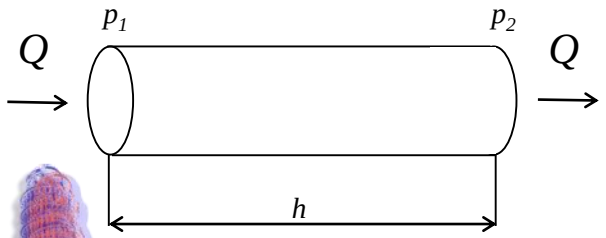


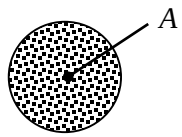


Darcy –Brinkman equations: enable calculation of average flow rate and shear in porous media, correlating pore level flow effects to the bulk fluid motion. In Darcy's model, the average fluid velocity depends on the permeability and the pressure gradient , so the tissue is seen as a continuum with a certain resistance to flow rather than an architectured mesh.

$$\bar{\tau} = \frac{\mu Q}{A\sqrt{K_p}}$$

$$K_p = \frac{\mu Q}{A\Delta p} h$$





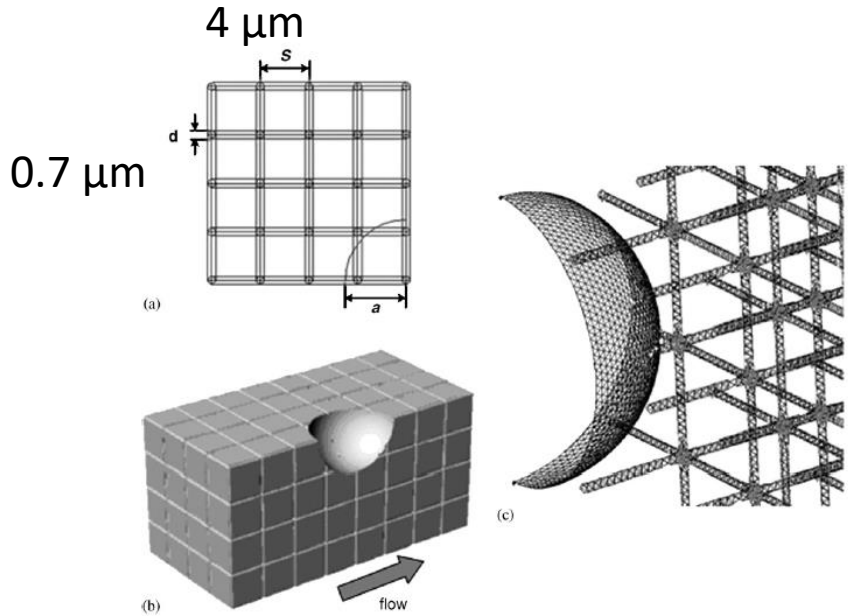
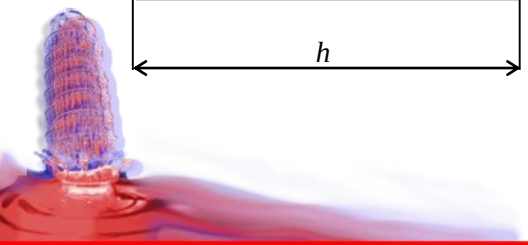


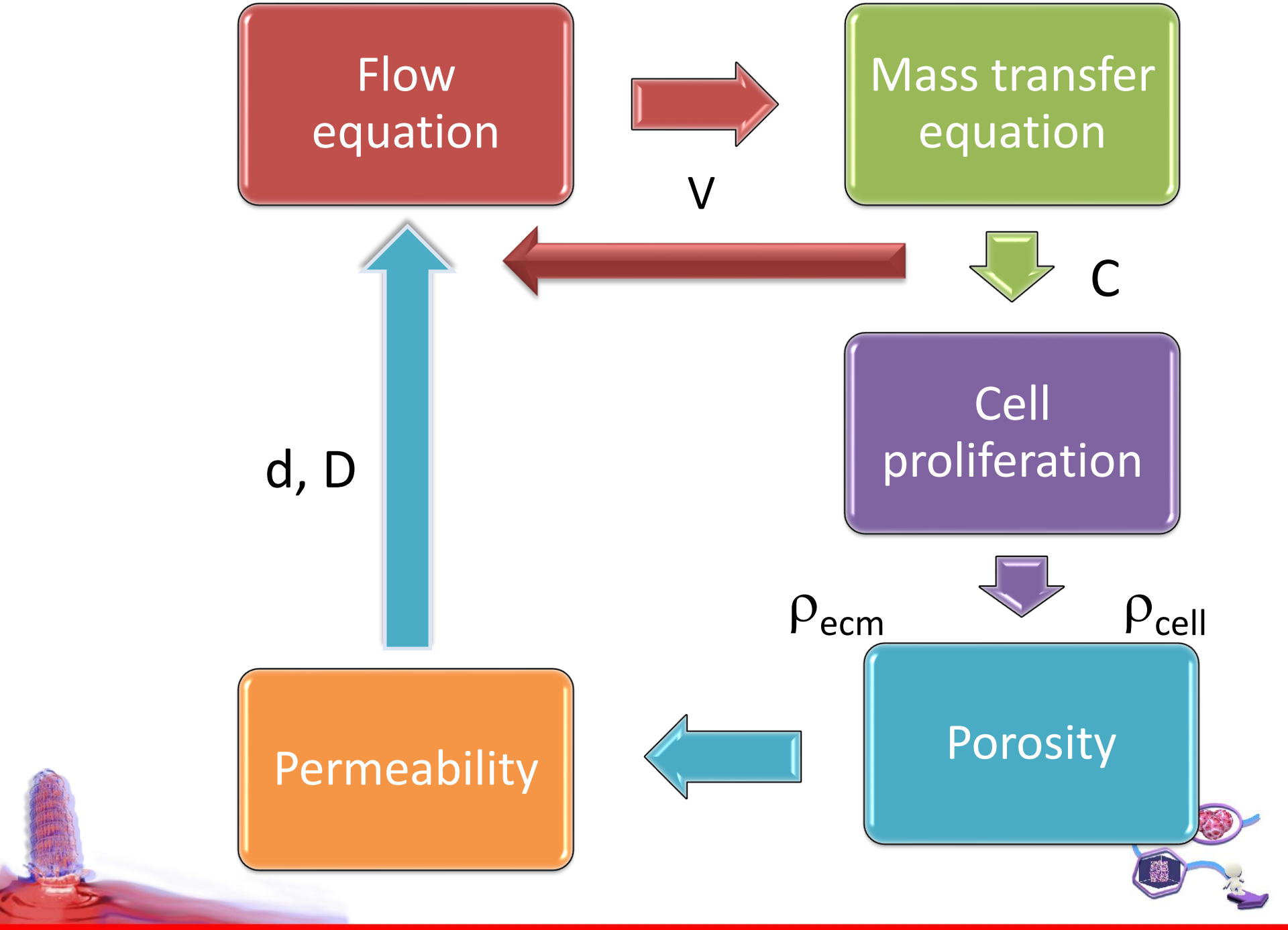
Fig. 1. CFD model setup. (a) Definition of the geometrical parameters of the model. (b) 3D rendering of the flow domain. (c) Detail of the mesh on the cell surface and nearby fibers.



The Brinkman correction to Darcys' equation takes into account the no slip condition at the walls of pores. (Darcy computes the pressure and the Brinkman correction computes the resulting velocity field as a function of fluid viscosity and permeability.)

$$q = \frac{-k}{\mu} \nabla p$$

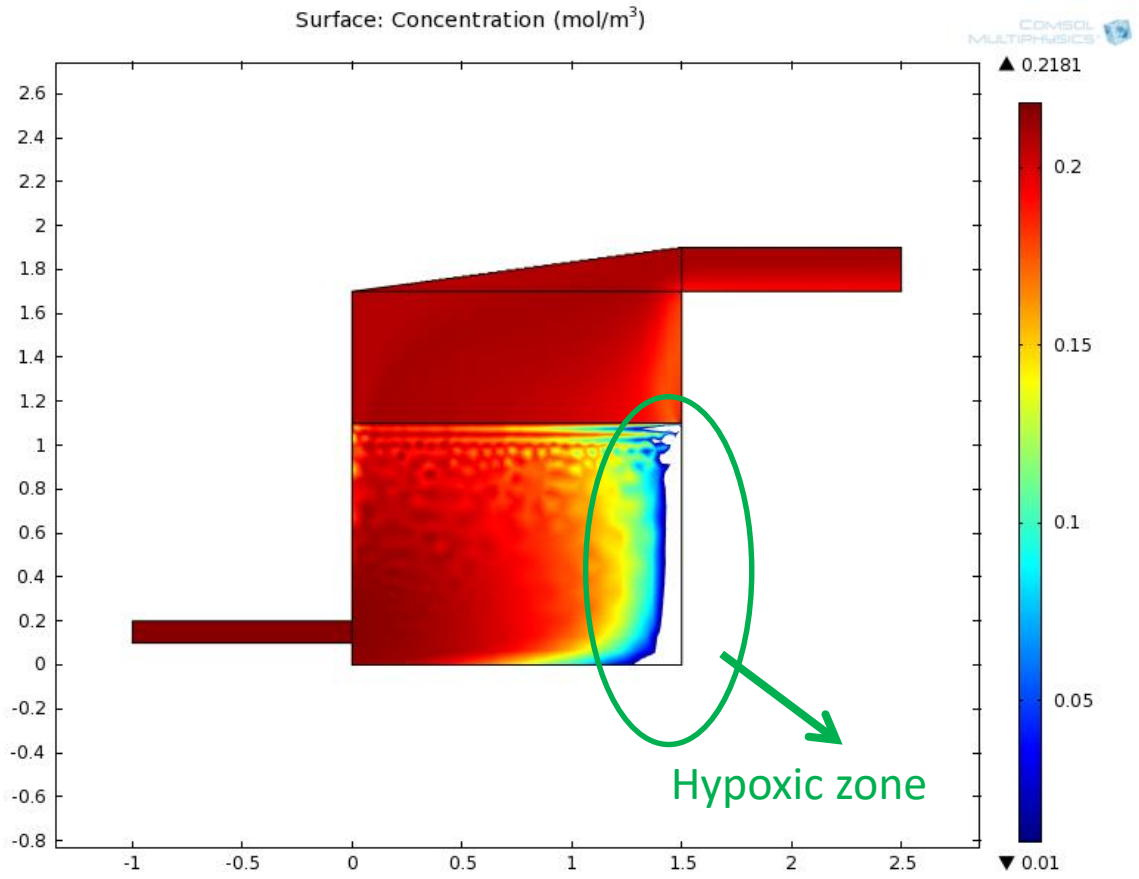
$$\mu \nabla^2 u + u = -k \nabla p$$


Flow equation
V
Mass transfer equation
C
Cell proliferation
ρ_{ecm}
ρ_{cell}
Porosity
Permeability
d, D

Oxygen consumption

Simulation

Adding reaction, and diffusion, convection multiphysics.
Sponge seeded with hepatocytes.



Reaction type Zero => constant consumption

Cell Density => $2.5.10^{-6}$ cells/cm^3

Hypoxic limit for hepatociye => 0.01 mol/m^3

OCR	Km	C crit	Co	D in water	D in sphere
1.10^{-18} to 1.10^{-16} moles.cell^{-1}.s^{-1}	1.10^{-3} mM (0.7 mmHg)	1.10^{-4} mM (0.07 mmHg)	0.2 mM	3.10^{-9} m^2.s^{-1}	2.10^{-9} m^2.s^{-1}
Medium height	δ (Heaviside)	Cell density in vivo	Vmax	Flow rates	
1 mm	flc1hs(c-0.02,0.0001)	$8.8.10^{14}$ cells·m^{-3}	0.034 mM.s^{-1}	10 to 500 µL/min	

Cell density in body: 1.10^9 cells/mL
Cell density in vitro: 1.10^6 cells/mL