Università di Pisa

Centro E. Piaggio
bioengineering and robotics research center

INGEGNERIA DEI TESSUTI BIOLOGICI:

STRESS-STRAIN TEST

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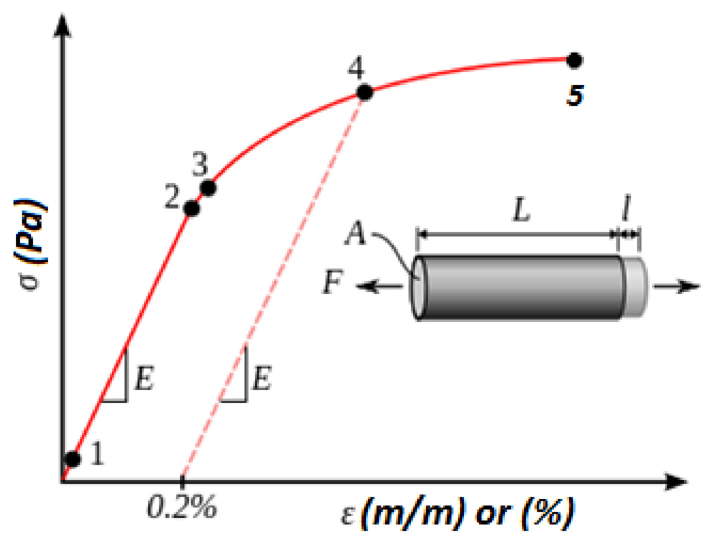
Stress-strain basics

Stress-Strain curve: *definition*

Stress-Strain curve is the relationship between the stress and the strain exhibited by a given material. It is *unique* for each material and is found by recording the amount of deformation (strain) at distinct intervals of tensile or compressive loading (stress). [Wikipedia]

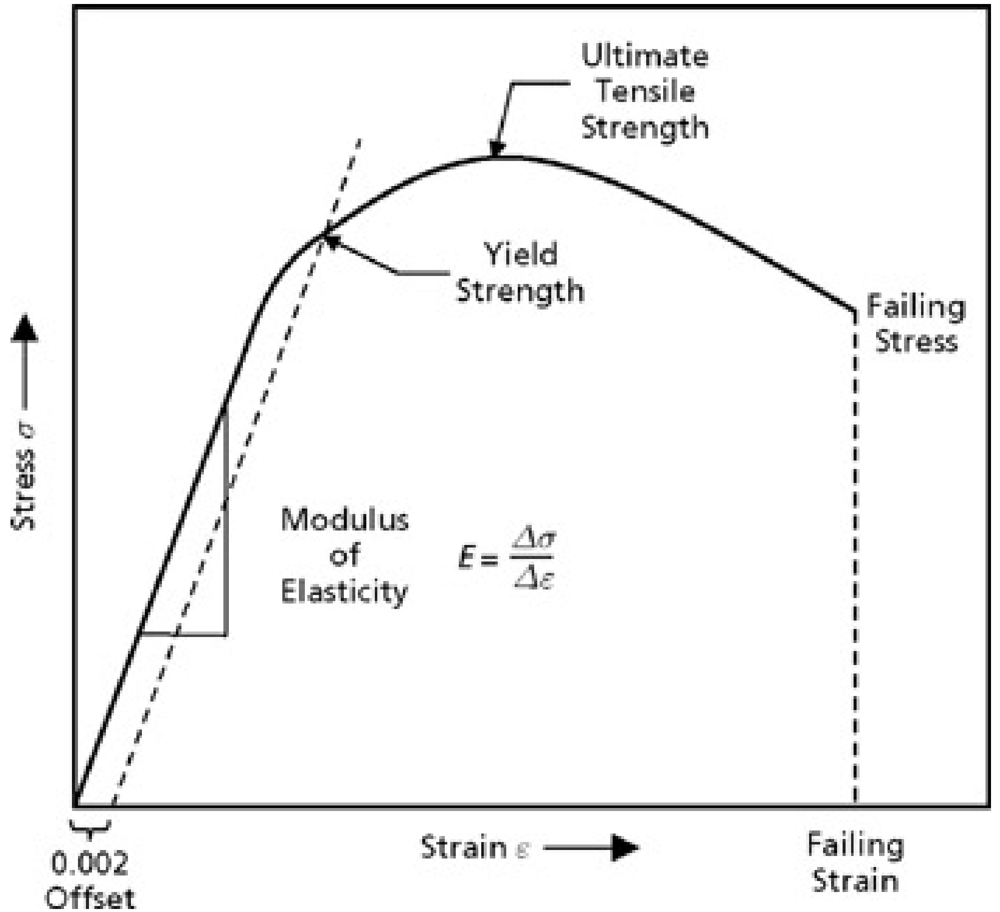
$$\sigma = \frac{F}{A}$$

$$\varepsilon = \frac{\Delta l}{l_0}$$



Stress-Strain curve: *utility*

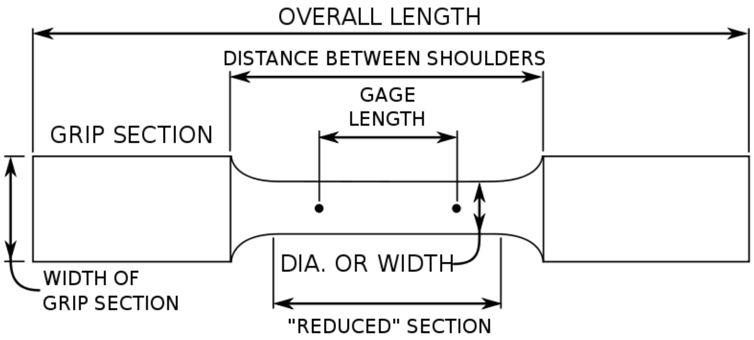
Evaluate material mechanical properties



Stress-Strain: *standard vs real sample*

Standard «dog-bone» shaped sample

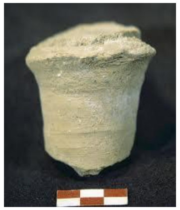
“…It has two shoulders and a gauge (section) in between. The shoulders are large so they can be readily gripped, whereas the gauge section has a smaller cross-section so that the deformation and failure can occur in this area…” [J. R. Davis, Tensile testing (2nd ed.), ASM International, 2004]



Real Sample

*Depends on what you are testing and typically it is **NOT standard**!*

- *No sufficient material*
- *Heterogeneous (especially tissues or natural materials)*



Obtaining experimental data

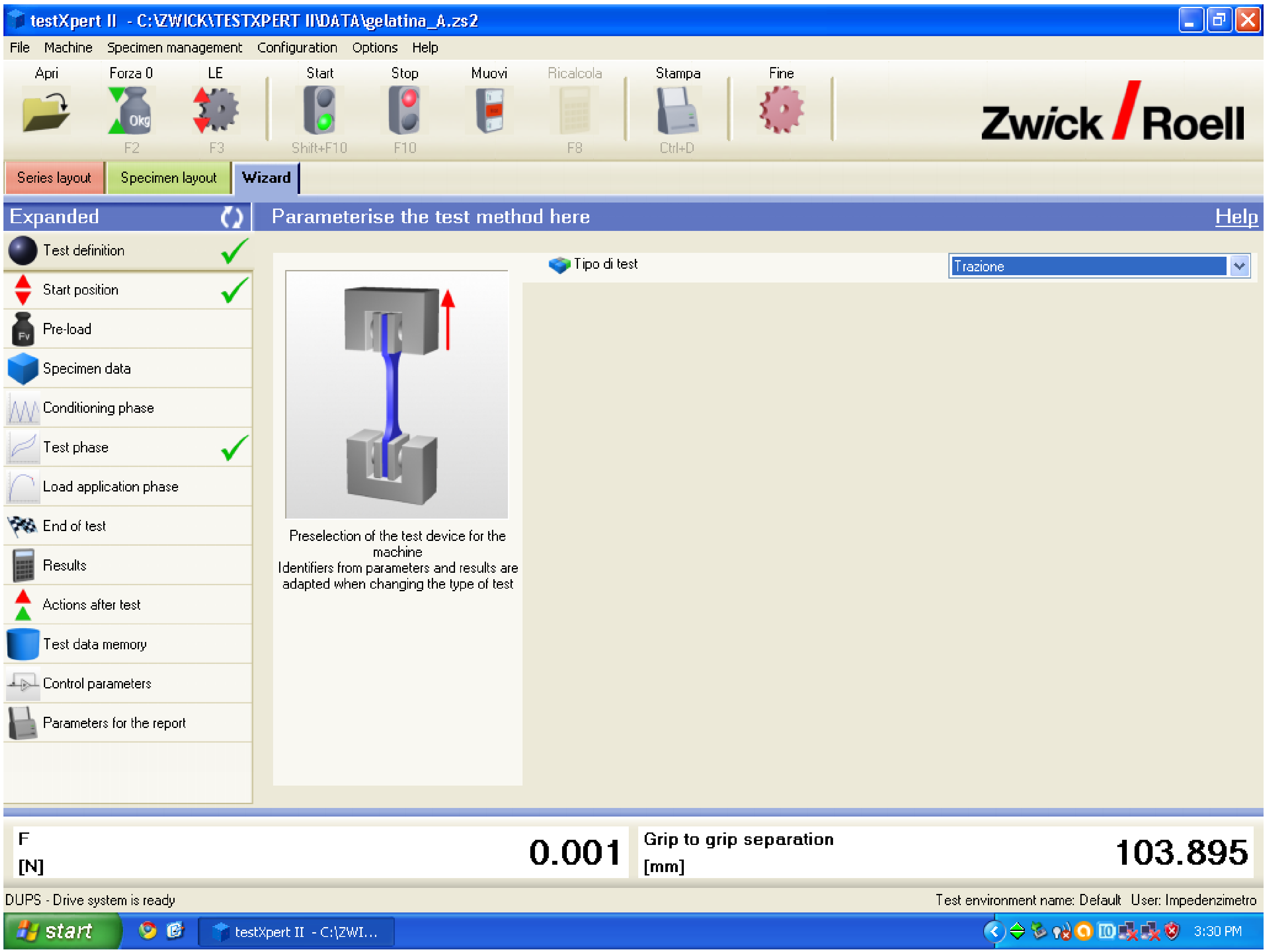
Zwick/Roell ProLine Z005

- **Uniaxial testing device**
- **Precision crosshead guidance**
- **Easy change of application:**
  - flexible plug and T-slot system allows **specimen grips/test fixtures** to be changed enabling a wide range of tests to be performed with the same testing machine
  - **load cell can be changed** depending on the required force resolution

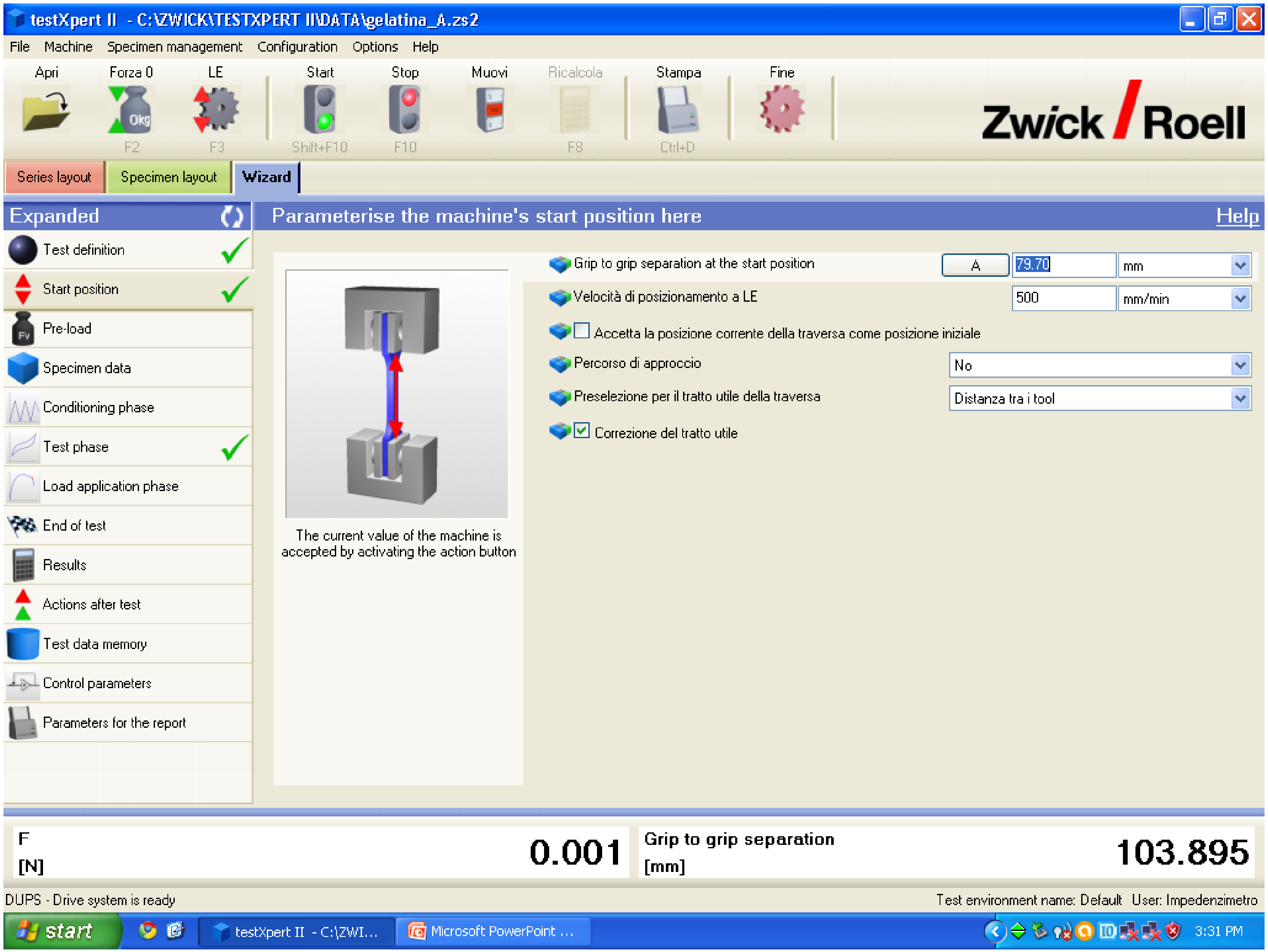
Zwick/Roell ProLine Z005

- ProLine is available with **test speeds from 0.0005 to 1500 mm/min,** depending on type. Testing machine speed is independent of the test load.
- The **high test-speed range can be used without restriction.** In addition, **test loads up to 110% of machine nominal load are permissible** to **compensate for heavy combinations** of test fixtures, accessories etc.
- ProLine is available for **test loads up to 100 kN** and with **test area heights from 1050 mm to 1450 mm.**
- ProLine can be **operated with standard commercial PCs or laptops** and **requires no special expansion card.**

TestXpert II software: wizard



TestXpert II software: wizard



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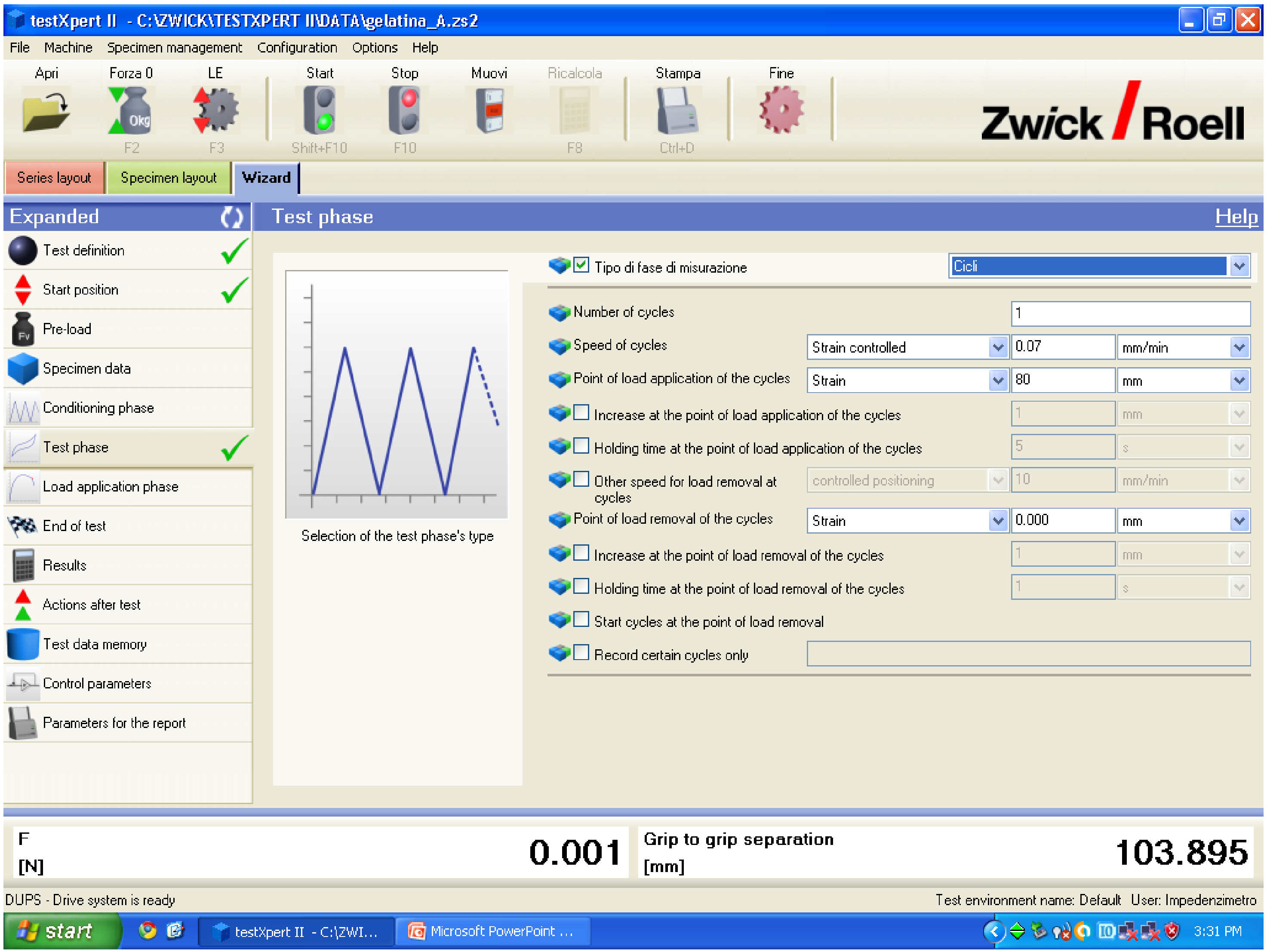
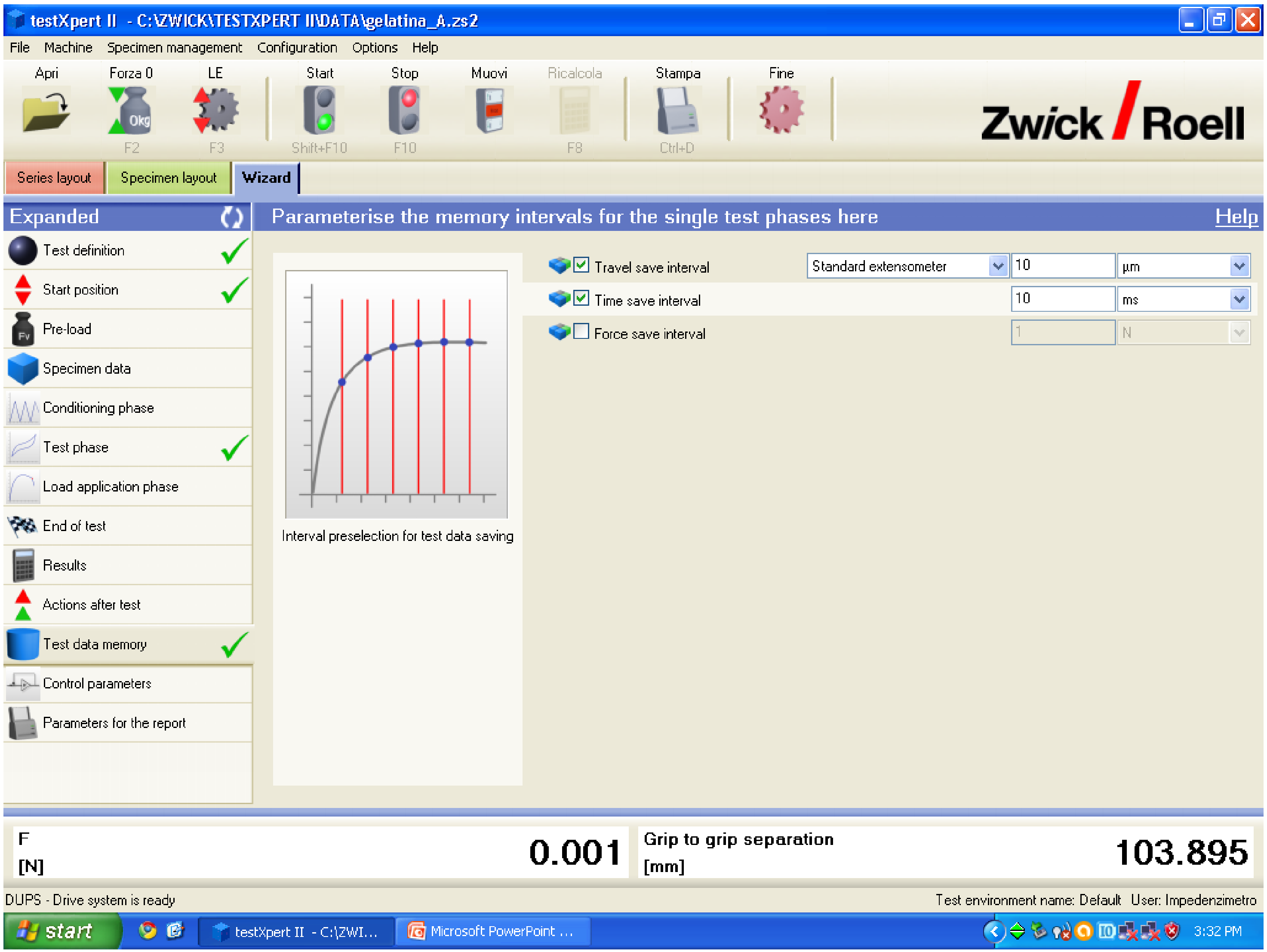
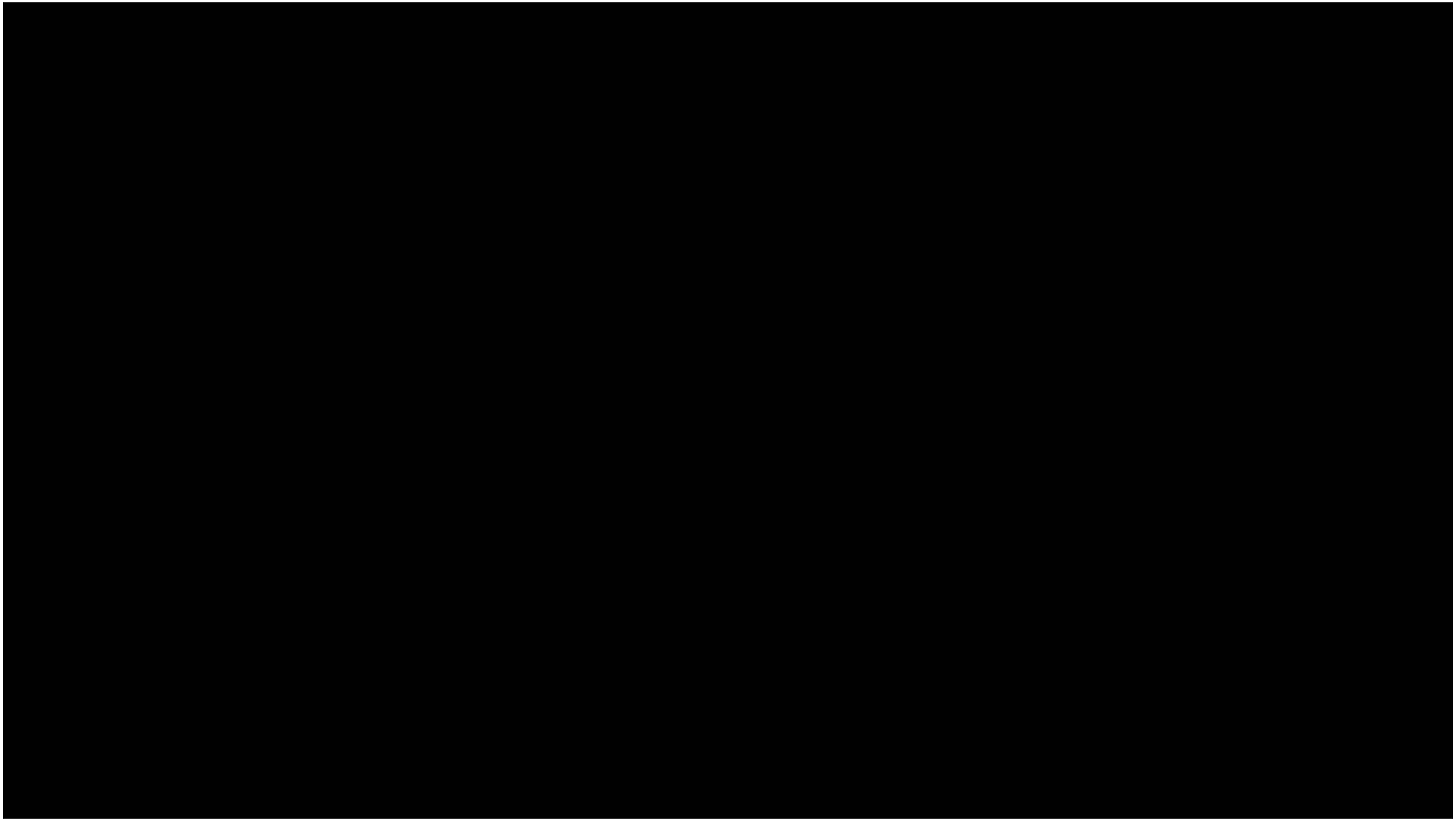


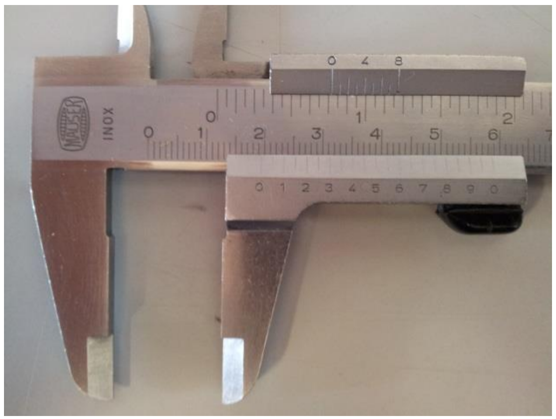
TestXpert II software: wizard



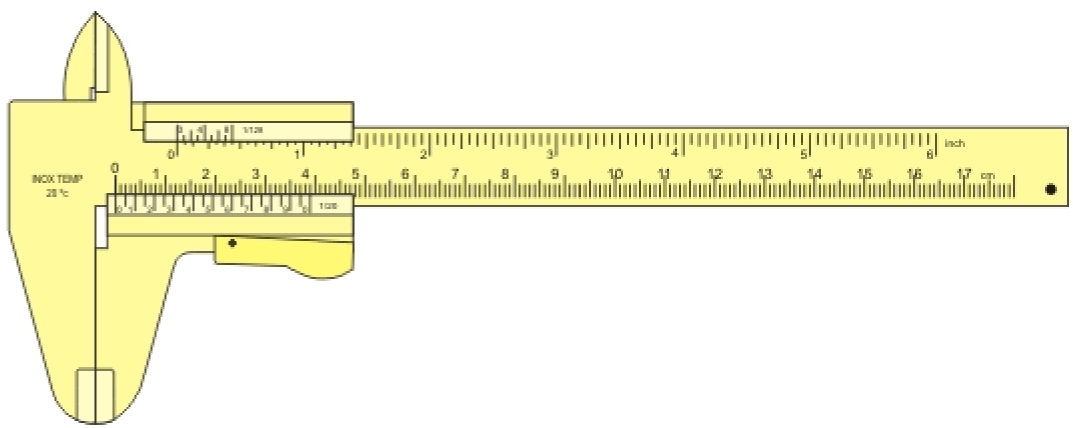
Zwick/Roell ProLine Z005



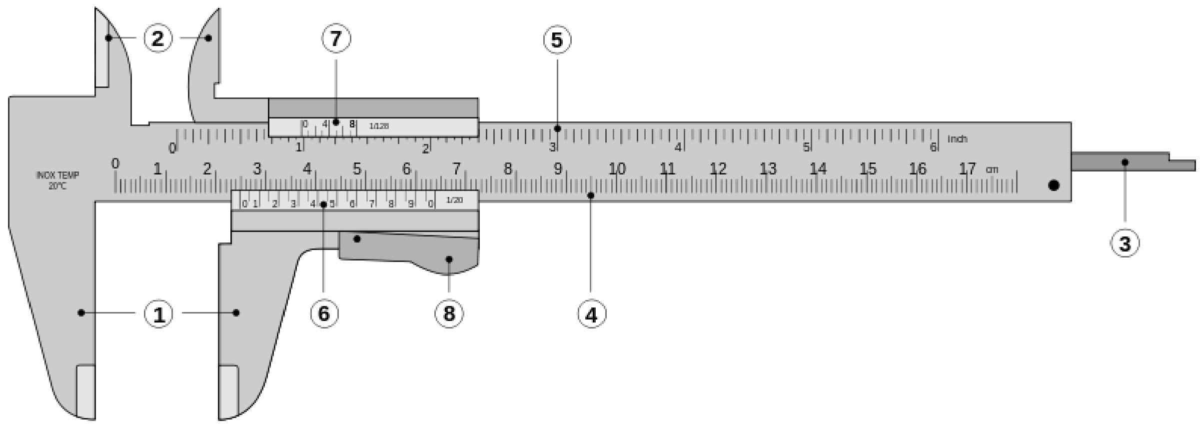
Calliper: *how to use it*



- direct reading of the distance measured with high accuracy and precision
- 0.05 mm resolution



Calliper: *how to use it*



1. Outside jaws: used to measure external diameter or width of an object

2. Inside jaws: used to measure internal diameter of an object

3. Depth probe: used to measure depths of an object or a hole

4. Main scale: scale marked every mm

5. Main scale: scale marked in inches and fractions

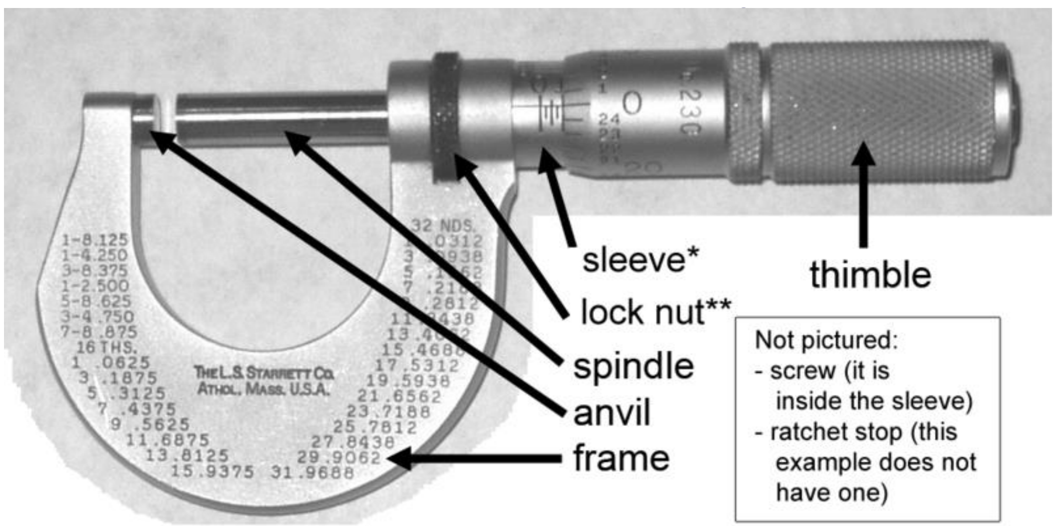
6. Vernier scale gives interpolated measurements to 0.1 mm or better

7. Vernier scale gives interpolated measurements in fractions of an inch

8. Retainer: used to block movable part to allow the easy transferring of a measurement

Micrometer: *how to use it*

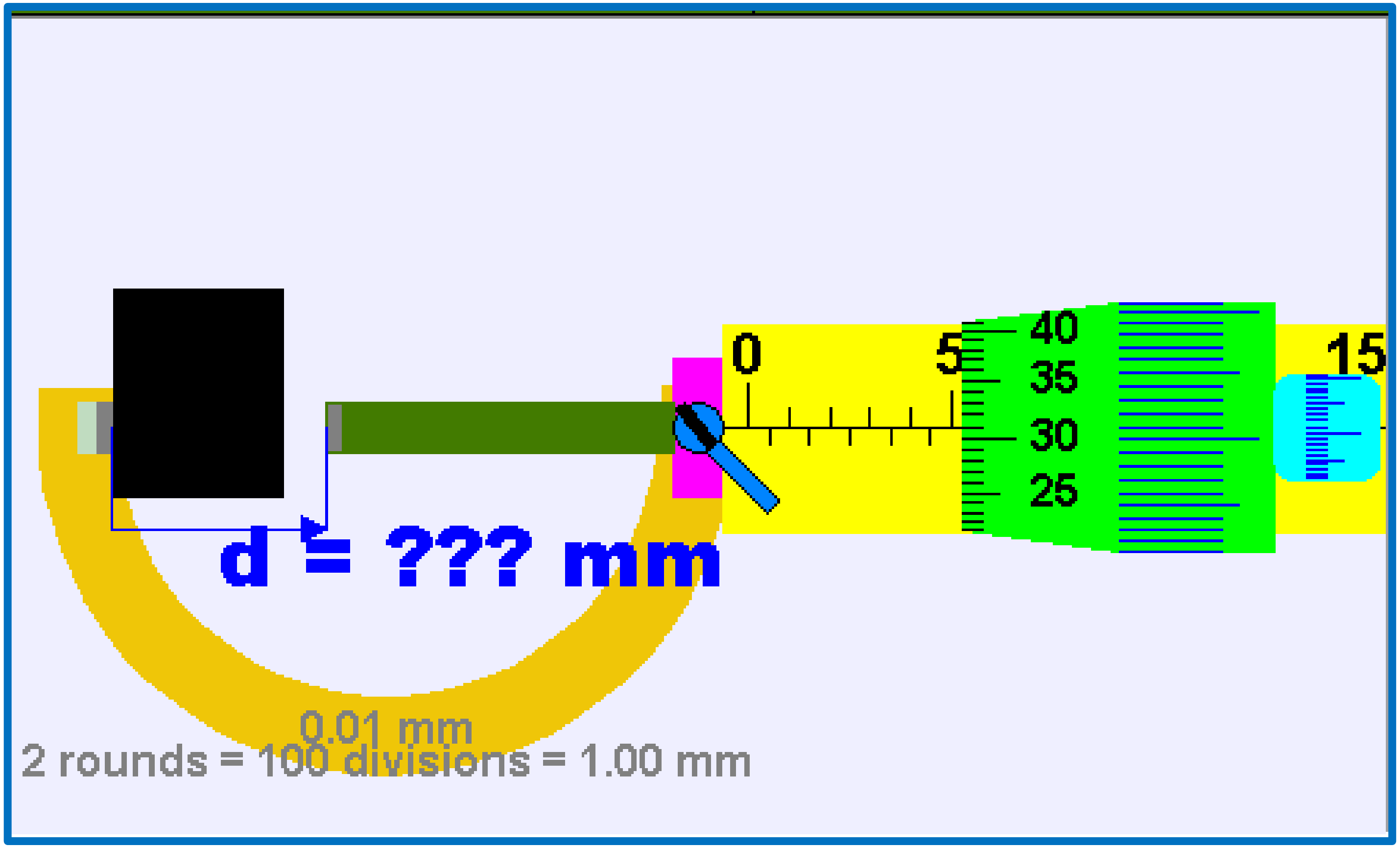
- Micrometers use the principle of a screw to amplify small distances (that are too small to measure directly) into large rotations of the screw that are big enough to read



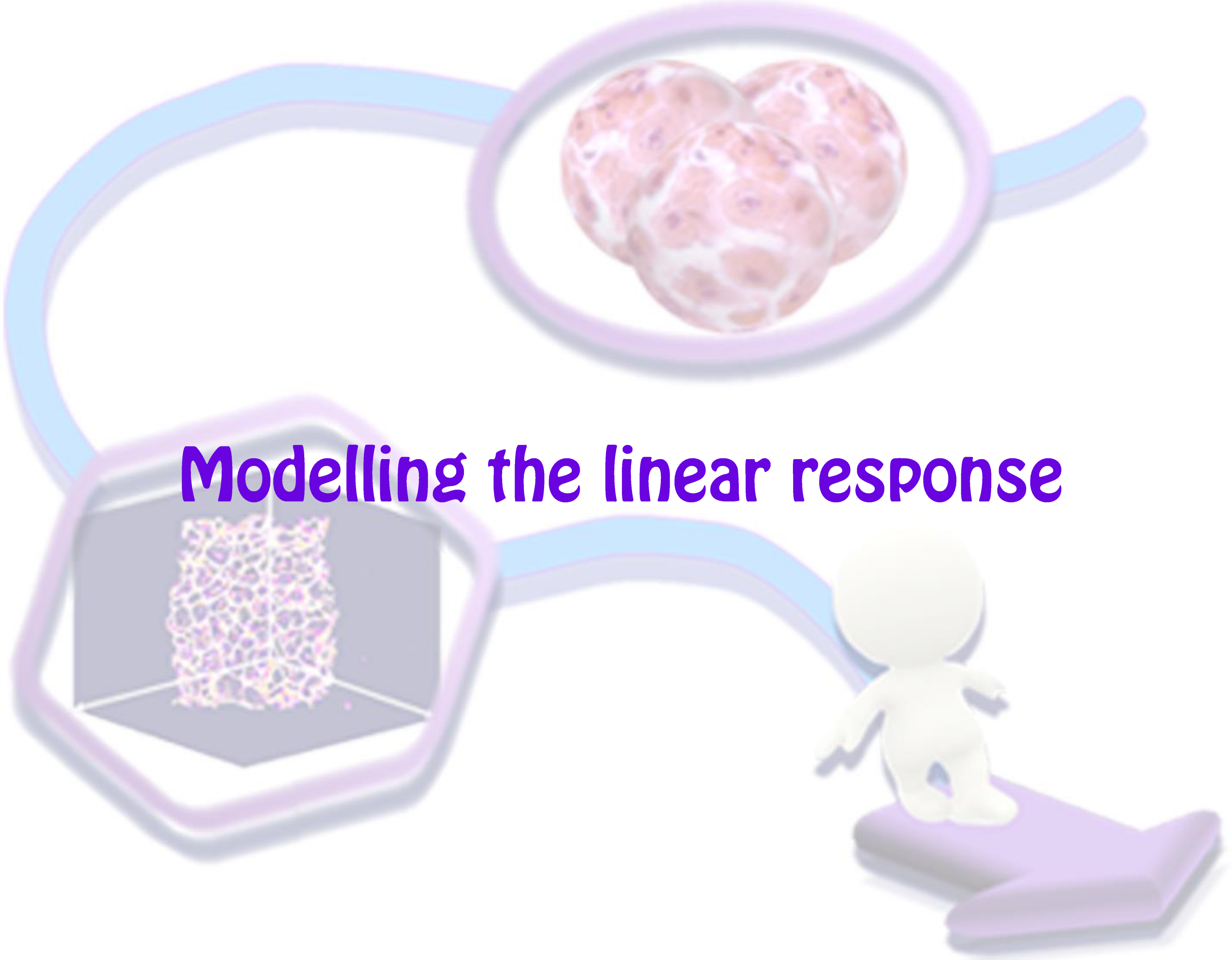
Sleeve is the most prevalent name. May also be called the *barrel* or *stock*.
**Aka *lock-ring*. Some mics have a *lock lever* instead.

- Resolution 0.01mm (10µm)

Micrometer: *how to use it*



Modelling the linear response



Linear Regression: *Definition*

In statistics, **linear regression** is an approach to modeling the ***relationship*** between a dependent variable y and one or more independent variables denoted x.

In linear regression, data are modeled using linear predictor functions, and unknown model parameters are estimated from the data.

Linear Regression

Relationship between ***input*** and ***output*** is assumed as:

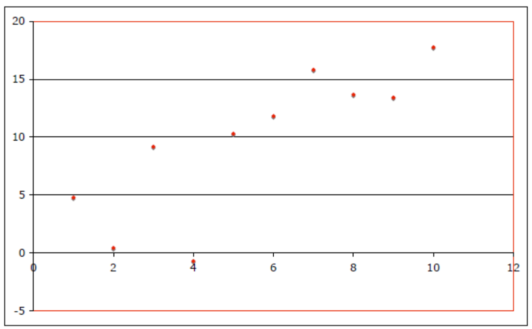
$$y = \alpha + \beta \cdot x + e$$

where

- ***y*** is the experimental output observed in response of an input ***x***
- ***α*** and ***β*** are the unknown parameters to estimate (i.e. intercept and slope of the linear fit)
- ***e*** is a random error term such that $E\{\varepsilon_i\} = 0 \quad \sigma^2\{\varepsilon_i\} = \sigma^2 \quad \sigma\{\varepsilon_i, \varepsilon_j\} = 0 \quad \forall\, i, j \ni i \neq j$

Linear Regression

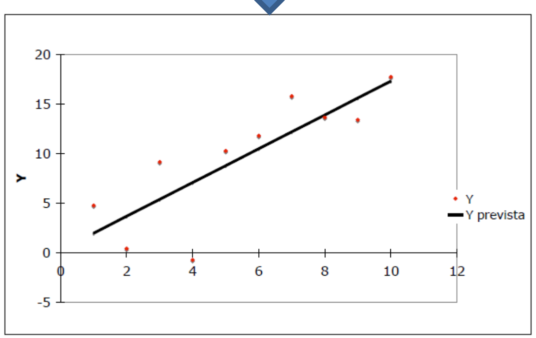
Parameters are estimated by minimizing the of ***sum of squared residual*** (SS_R)



$$y = \alpha + \beta \cdot x$$

$$SS_R = \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

residual = vertical distance between real data and estimated curve

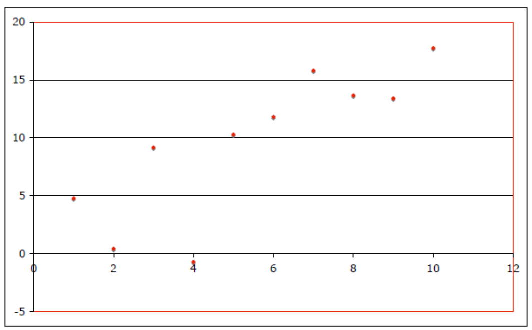


Assumptions:

- (x_i, y_i) are independent and identically distributed observations
- x_i are random and sampled together with y_i

Linear Regression

Parameters are estimated by minimizing the of ***sum of squared residual*** (SS_R)



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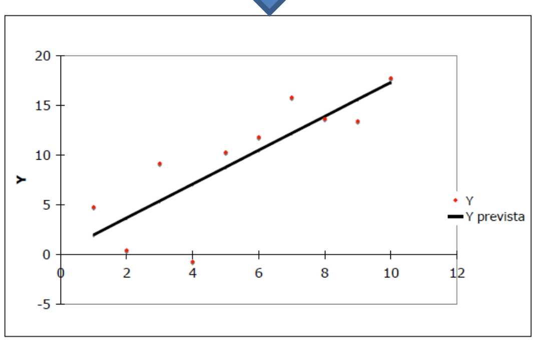


The SS_R is an index of inherent variability, quantifying how much the fitted line differs from the real output due the error (**e**)

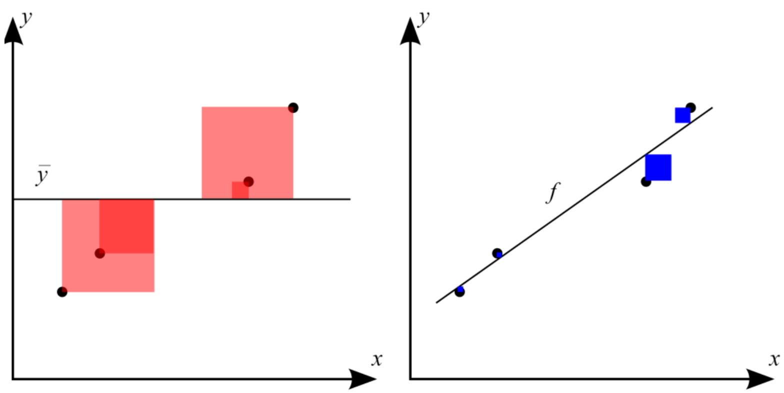
Linear Regression

R^2: a measure of goodness-of-fit of linear regression

$$R^2 := 1 - \frac{SS_R}{S_{yy}}$$

The coefficient of determination (R^2) ranges from 0 (model does not fit the data) to 1 (perfect fit)

$$SS_R = \sum_{i=1}^{n} (y_i - a - bx_i)^2 \qquad S_{yy} = \sum_i (y_i - \bar{y})^2$$



The **better the linear regression** (on the right) fits the data in comparison to the simple average (on the left), the **closer the value of R^2 is to 1**. The **areas of** the **blue squares** represent **SS_R**. The **areas of** the **red squares** represent the **S_{yy}**.

Linear Regression

R^2: a measure of goodness-of-fit of linear regression

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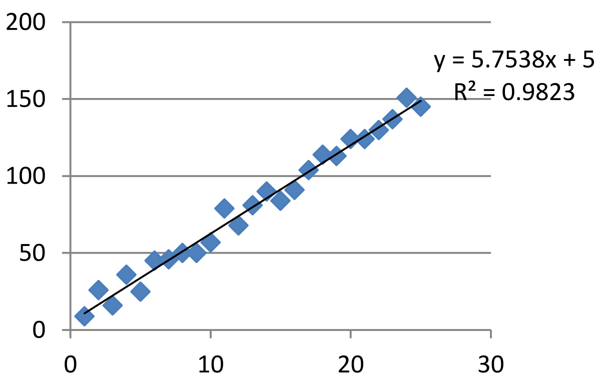
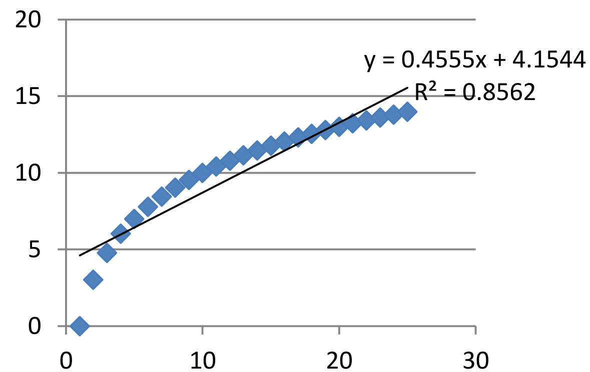
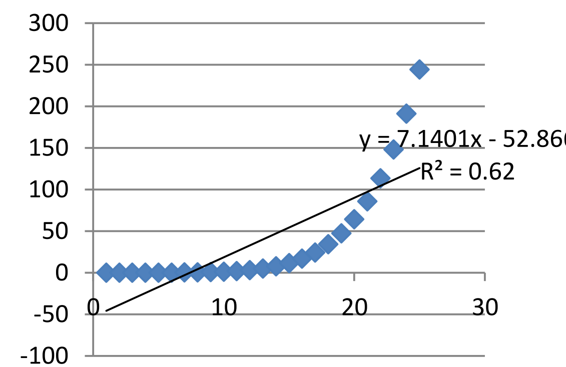


Good fit
($R^2 > 0.9$)



Poor fit
($0.9 > R^2 > 0.8$)

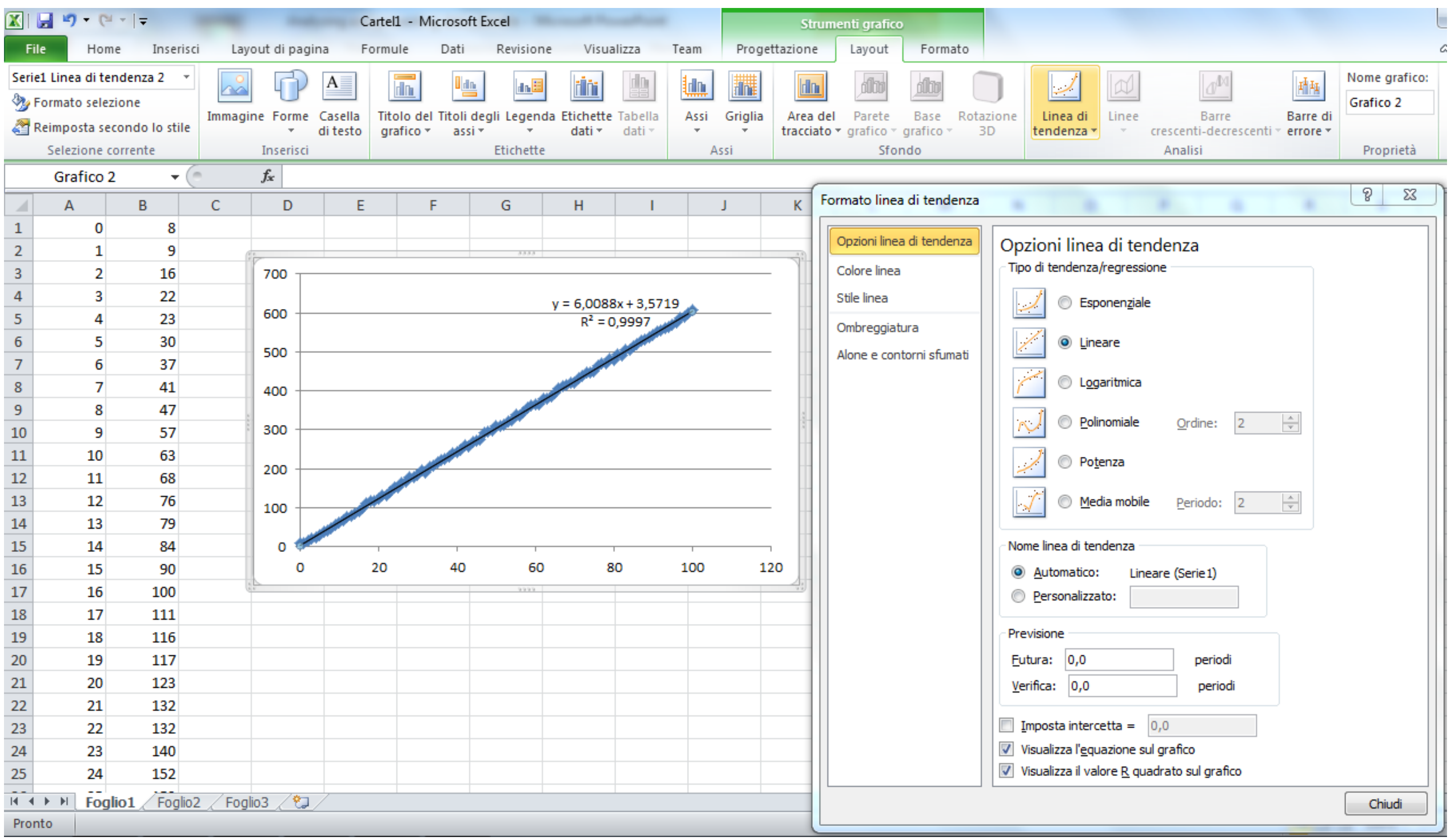


Very bad fit
($R^2 < 0.8$)

Microsoft Excel: *Linear Regression*

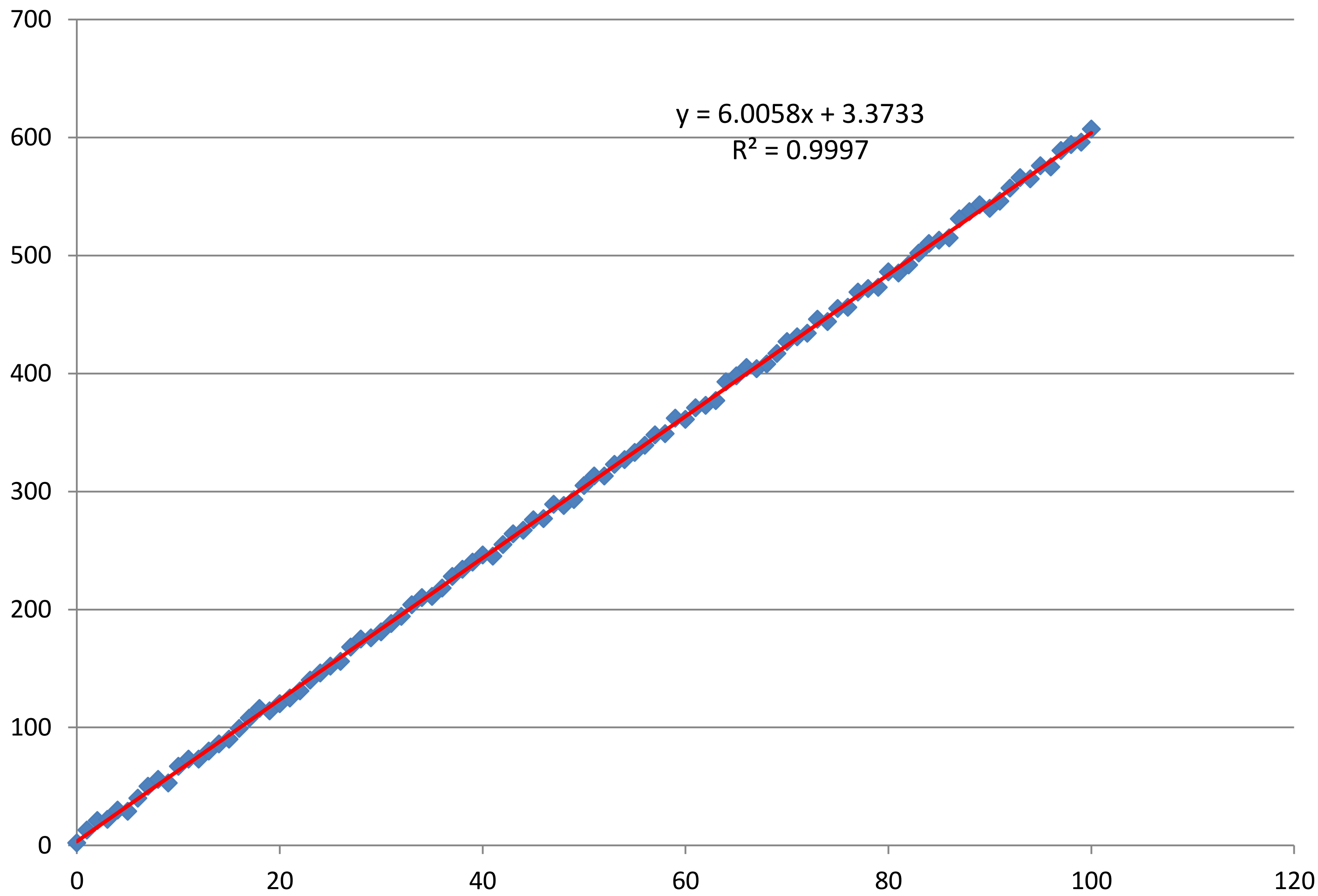
Two ways to evaluate fit parameters

- *Directly on the plot*: add trendline



- Easy method
- Data on plot
- GUI help

Microsoft Excel: *Linear Regression*



Microsoft Excel: *Linear Regression*

Two ways to evaluate fit parameter

- *As a cell function*: use *linear estimation* function

Known_y's: experimental y values you wish to fit using y = ax + b

Const: a logical value to fix line intercept to 0 (const =0 → b=0)

In Italian...

=REGR.LIN(known_y's, known_x's, const, stats)

Known_x's: experimental x values you wish to fit using y = ax + b

Stats: a logical value to specify whether to return additional regression statistics (stats = 1 returns additional regr. statistics)

	A	B	C	D	E	F
1	m_n	m_{n-1}	...	m_2	m_1	b
2	se_n	se_{n-1}	...	se_2	se_1	se_b
3	r_2	se_y				
4	F	d_f				
5	ss_{reg}	ss_{resid}				

Arrangement of fitted parameters in Excel (*select the necessary number of cells then press CRTL+SHIFT+ENTER to enter an array formula)*

Case of study: the hair

Typical experiment and analysis

Testing the hair: experimental details

1. Measure **hair diameter** using the **micrometre** to evaluate the sample **cross sectional area (**A**)**
2. **Clamp the hair using acetate sheets**, then measure the **distance between the latter** using the **calliper** to evaluate **sample initial length (**$\boldsymbol{l_0}$**)**
3. Assemble the **testing setup**
4. Acquire **force-displacement** curves at a **0.01 s**$^{-1}$ **strain rate** using the Zwick/Roell ProLine Z005 testing machine
6. **Identify the first loading point**, then **offset force and displacement vectors** to start from 0 in correspondence of the initial loading point and **convert them** into **stress (**$\boldsymbol{\sigma = F/A}$**)** and **strain (**$\boldsymbol{\varepsilon = \Delta l/l_0}$**)**, respectively
7. **Plot stress-strain curve** of the hair
8. Evaluate the **elastic modulus of the hair** as the **slope of the stress-strain curve** in the **first linear (elastic) tract**

Practical exp: hair mechanical test

When: 6 Apr 2016 – 9.00-13.00

Where: Laboratori Centro di Ricerca “E. Piaggio” – 3° piano Polo A, Scuola di Ingegneria, Università di Pisa

Contacts

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