Esercizio 1 (1)

Il Riunatore indolo nanoparticelle de ~~[illegible]~~ si bloccano negli alveoli e riducono il passaggio di gas quindi ovvio nel modello dei 2 strati un termine aggiuntivo

$$W^{+} = -D_{part} \frac{\Delta P}{\delta_{part}} K_{part} A_{part}$$

Inoltre la presenza di monossido altera la reazione col gruppo eme e quindi varia anche il termine di legge de rene

$$W = \frac{1}{K([Hb]-[CO])} \Delta P$$

2) Considero il modello di ossigenazione del globulo rosso tenendo conto che il monossido si lega ~~irreversibilmente~~ all'ossigeno

$$\begin{cases} \frac{\partial c'}{\partial t} = D_2 \frac{\partial^2 c'}{\partial x^2} \\ \frac{\partial c}{\partial t} = D_1 \frac{\partial^2 c}{\partial x^2} + K'(y_0 - y) - Kcy \\ \frac{\partial y}{\partial t} = D_{Hb} \frac{\partial^2 y}{\partial x^2} + K'(y_0 - y) - Kcy \end{cases}$$

~~Re spunnode~~
Considero
$D_{Hb} = \phi$
$K' = \phi$

Caso più semplice considero il sistema temporalmente.

$$\begin{cases} D_2 \dfrac{\partial^2 c'}{\partial x^2} = \phi \\ D_1 \dfrac{\partial^2 c}{\partial x^2} = +kcy \\ \dfrac{\partial y}{\partial t} = -kcy \end{cases}$$

$$\frac{\partial c'}{\partial x} = A \qquad c' = Ax + B \qquad c'(b_1 + b_2, \phi) = C_o$$

$$c'(b_1, \phi) = c_i$$

$$c' = \frac{(c_o - c_i)(x - b_1)}{b_2} + c_i$$

sapendo che $D_2 \dfrac{\partial c'}{\partial x} = \alpha D_1 \dfrac{\partial c}{\partial x}$.

$$\alpha D_1 \frac{\partial c}{\partial x} = \underbrace{D_2 \frac{(c_o - c_i)}{b_2}}_{= \Gamma}$$

$$\frac{\partial c}{\partial x} = \frac{\Gamma}{\alpha D_1} = \Delta$$

$$\frac{\partial c}{\partial x} = \Delta \qquad c = \Delta x + \varepsilon. \qquad c(0,0) = 0 \implies \varepsilon = \phi$$

$$c = \Delta x.$$

$$\frac{\partial y}{y} = -K_c dt$$

$$\ln y \Big|_{y}^{y_0} = -Kct$$

$$\ln \frac{y}{y_0} = -K_c t$$

$$y = y_0 e^{-K_c t} = y_0 e^{-k \Delta x t}$$

1. TEORIA

CLEARANCE = 0, x $U_x = \emptyset$ → RIASSORBIMENTO CHE → ALLORA COMPENSA TOTALMENTE LA FILTRAZIONE

CLEARANCE = VFG, NESSUN INTERVENTO DEL TUBULO → INULINA!

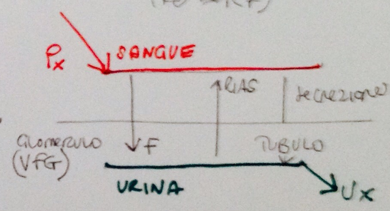
2. CLEARANCE INULINA = GFR

$$[i]_p \cdot VFG = [i]_u \dot{V}$$

~~[illegible]~~ TUTTO CIÒ CHE VIENE FILTRATO VIENE ESCRETO ($R_e = R_f$)

$$VFG = \frac{[i]_u \dot{V}}{[i]_p} = \frac{980}{10} \cdot 1.5 = 147 \frac{ml}{min}$$

$\dot{V} = \frac{180\ ml}{2\ h} = \frac{180\ ml}{2 \cdot 60\ min} = 1.5 \frac{ml}{min}$



3. $R_{TOT} = 60\ min/cm = R_D + R_B + R_m$

$K = 1/R_{TOT} = 0.016\ cm/min$

$A = 1\ m^2 = 10^4\ cm^2$

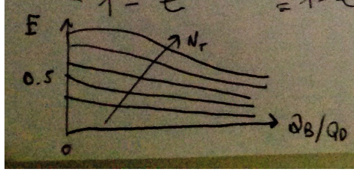
$C_{Bi} = 148\ mg/dl$

$$C_{Bo} = C_{Bi}\, e^{-KA/Q_B} = 148\ mg/dl\ e^{-\frac{0.016 \cdot 10^4}{147\ cm^3}\ min/m \cdot \frac{cm}{min} \cdot cm^2} = 148\ mg/dl \cdot 0.337 = 49.83\ mg/dl$$

$$E = 1 - \frac{C_{Bo}}{C_{Bi}} = 0.66$$

$$= 1 - e^{-KA/Q_B} = 1 - e^{-N_T}$$

PER AUMENTARE E DEVE AUMENTARE N_T.
$N_T = KA/Q_B$, QUINDI POSSO AGIRE O SU K O SU A, MA NON SU Q_B —

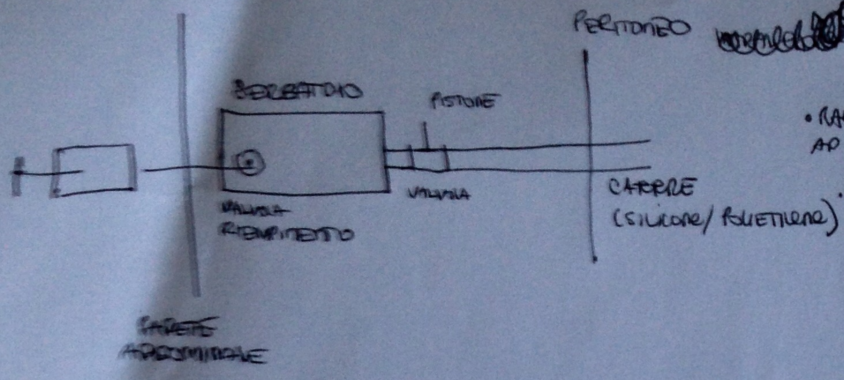


• SE DECIDO DI AGIRE SU A MANTENENDO K = cost

$$A = -\ln(1-0.80) \cdot \frac{Q_B}{K} = 1.47\ m^2$$

$$K = -\ln(1-0.80) \cdot Q_B/A = 0.023\ cm/min$$

ES_ PANCREAS



$\boxed{\text{2 POSSIBILI INFUSIONI}}$

PORTALE

- RAPIDA RISPOSTA AD AUMENTI DI GLUCOSIO
- RAPIDA INSULINIZ. EPATICA
- UNA EVENTUALE TROMBOSI CAUSEREBBE EMBOLIA NEL FEGATO

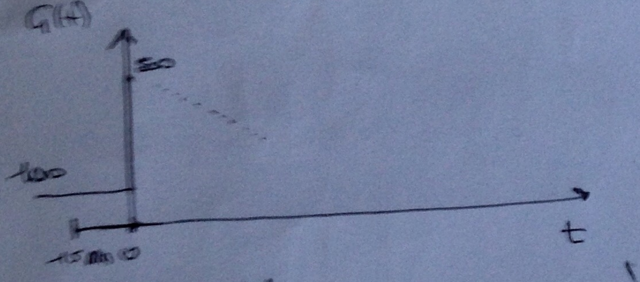
INTRAPERITONEALE

- INSULINA VIENE ASSORBITA DALLA VENA PORTA → LIVELLI INSULINA ≃ A QUELLI FISIOLOGICI

$$20\,g \longrightarrow \frac{20 \cdot 10^3\, mg}{50\, dl} = 400\, mg/dl$$

$$G(0) = 400\, mg/dl + 100\, mg/dl = 500\, mg/dl$$

$$I_{clemens}(0) = K \left.\frac{dG(t)}{dt}\right|_{dt = 15\, min} = 0.015 \frac{500-100}{15 \cdot 60} \cdot 10^3\, \mu U = 6.66\, \mu U/dl$$



$G(0) = 500\, mg/dl$

$G(15) = 500\, mg/dl - 1/4 \; 500\, mg/dl = 375\, mg/dl$

$G(30) = G(15) - 1/4\, G(15) = 281.25\, mg/dl$

$G(45) = G(30) - 1/4\, G(30) = 210.93\, mg/dl$

$G(60) = G(45) - 1/4(G(45)) = 158.22\, mg/dl$

$G(75) = G(60) - 1/4\, G(60) = 118.66$

$G(90) \cong G_{basale}$

$I(0) = 6.6\, \mu U/dl$

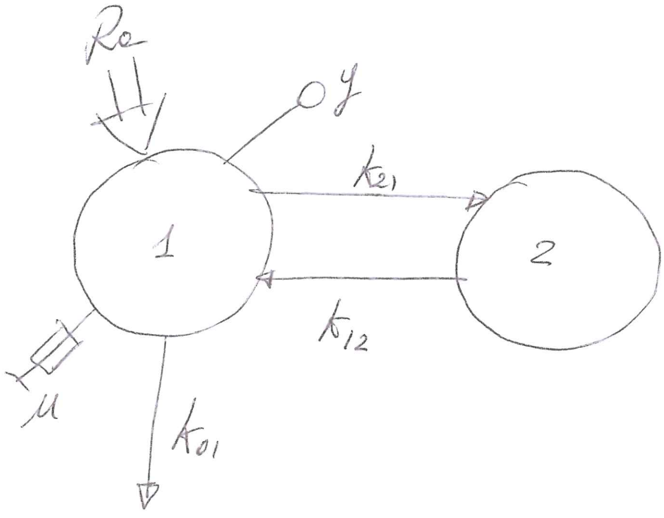
$I(15) = 4.995$

$I(30) = 3.74$

$I(45) = 2.80$

$I(60) = 2.09$

$I(75) = I_b \approx 2\, \mu U/dl$



Equazioni del [illegible]

$$\begin{cases} \dot{q}_1 = -(k_{01} + k_{21})\, q_1 + k_{12}\, q_2 + u \\ \dot{q}_2 = + k_{21}\, q_1 - k_{21}\, q_2 \\ y = \dfrac{q_1}{V_1} \end{cases}$$

Trasformata di Laplace $\{q_i\} \overset{\mathcal{L}}{=} \{Q_i\}$

$$\begin{cases} sQ_1 = -(k_{01} + k_{21})\, Q_1 + k_{12}\, Q_2 + U \\ sQ_2 = + k_{21}\, Q_1 - k_{12}\, Q_2 \\ Y = \dfrac{Q_1}{V_1} \end{cases}$$

$$A = \begin{bmatrix} -(k_{01} + k_{21}) & k_{12} \\ k_{21} & -k_{12} \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \dfrac{1}{V_1} & 0 \end{bmatrix}$$

$$H = C\,(sI - A)^{-1} B$$

$$= \begin{bmatrix} \frac{1}{V_1} & 0 \end{bmatrix} \underbrace{\begin{bmatrix} s + k_{01} + k_{21} & -k_{12} \\ -k_{21} & s + k_{12} \end{bmatrix}}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathrm{Det} = (s + k_{12})(s + k_{01} + k_{21}) - k_{12} k_{21}$$

$$s^2 + s(k_{12} + k_{01} + k_{21}) + k_{12} k_{01}$$

$$\begin{bmatrix} \frac{s + k_{12}}{\mathrm{Det}} & \frac{+k_{12}}{\mathrm{Det}} \\ \frac{k_{21}}{\mathrm{Det}} & \frac{s + k_{01} + k_{21}}{\mathrm{Det}} \end{bmatrix}$$

$$H = \frac{1}{V_1} \; \frac{s + k_{12}}{s^2 + s(k_{12} + k_{01} + k_{21}) + k_{12} k_{01}}$$

→ La funzione di trasferimento è della forma

$$H = \frac{\beta_2 s + \beta_1}{s^2 + \alpha_2 s + \alpha_1}$$

Sommario esaustivo

$$\beta_2 = \frac{1}{V_1}$$

$$\beta_1 = \frac{k_{12}}{V_1}$$

$$\alpha_2 = k_{12} + k_{01} + k_{21}$$

$$\alpha_1 = k_{12} k_{01}$$

4 parametri

$[V_1, k_{12}, k_{01}, k_{21}]$

$\rightarrow$ matrice della funzione di trasferimento G

$$\begin{array}{c} \\ \beta_2 \\ \beta_1 \\ \alpha_2 \\ \alpha_1 \end{array}
\begin{array}{c}
\begin{array}{cccc} V_1 & k_{12} & k_{01} & k_{21} \end{array} \\
\begin{bmatrix} -\frac{1}{V_1^2} & 0 & 0 & 0 \\ -\frac{k_{12}}{V_1^2} & \frac{1}{V_1} & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & k_{01} & k_{12} & 0 \end{bmatrix}
\end{array}$$

$$\mathrm{Det}(G) = \left(-\frac{1}{V_1^2}\right) \cdot \frac{1}{V_1} \left(-k_{12}\right) = \frac{k_{12}}{V_1^3}$$

$\mathrm{Rank}(G) = 4 = \#$ parametri $\Rightarrow$ MODELLO IDENTIFICABILE UNIVOCAMENTE

Uscita sperimentale

$$y(t) = 3.60\, e^{-0.08t} + 2.40\, e^{-0.04\, t}$$

$$y(t) \overset{\mathcal{L}}{=} Y(s)$$

$$\longrightarrow Y(s) = \frac{3.60}{s+0.08} + \frac{2.40}{s+0.04}$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1.80}{s+0.08} + \frac{1.20}{s+0.04}$$

$$H(s) = \frac{1.80 * (s+0.04) + 1.20\,(s+0.08)}{(s+0.08)(s+0.04)}$$

$$= \frac{3 \cdot s + 0.168}{s^2 + 0.12\, s + 0.0032}$$

Inserisco i valori numerici di α_i e β_i nel sommario esaustivo

$$\begin{cases} 3 = \dfrac{1}{V_1} \\ 0.168 = \dfrac{k_{12}}{V_1} \\ 0.12 = k_{12} + k_{01} + k_{21} \\ 0.0032 = k_{12}\, k_{01} \end{cases}$$

$$\longrightarrow V_1 = \frac{1}{3} = 0.33$$

$$k_{12} = \frac{0.168}{3} = 0.056$$

$$k_{01} = \frac{0.0032}{0.056} = 0.057$$

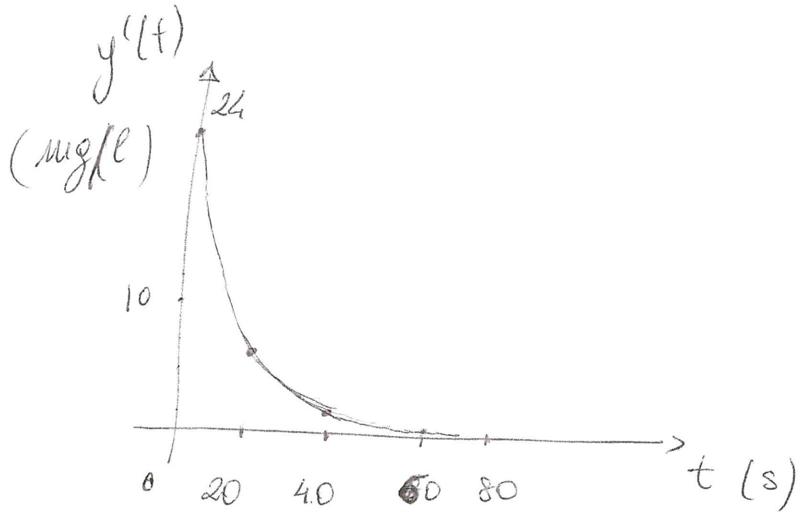
$$k_{21} = 0.12 - 0.056 - 0.057 = 0.007$$

Se il bolo è 8 mg, l'uscita nel dominio di Laplace diventa

$$Y'(s) = H(s) \cdot U'(s)$$

$$= \frac{1.80 * 8}{s+0.08} + \frac{1.20*8}{s+0.04} = \frac{14.4}{s+0.08} + \frac{9.6}{s+0.04}$$

$$\Rightarrow y'(t) = \mathcal{L}^{-1}\{Y'(s)\} = 14.4\, e^{-0.08t} + 9.6\, e^{-0.04t}$$



$y'(0) = 24$ mg/l

$y'(20) = 7.2$ mg/l

$y'(40) = 2.5$ mg/l

$y'(60) = 0.99$ mg/l

$$y'(t^*) = \frac{y'(0)}{2} = 12 \text{ mg/l}$$

$t^* = ?$ sostituisco $e^{-\lambda_2 t^*}$

$$e^{-0.04\,t^*} = x \quad \rightarrow \quad e^{-0.08\,t^*} = x^2$$

La funzione di trasferimento diventa

$$12 = 14.4\,x^2 + 9.6\,x \quad ; \quad 14.4\,x^2 + 9.6\,x - 12 = 0$$

$$x = \frac{-9.6 \pm \sqrt{9.6^2 + 4 * 14.4 * 12}}{14.4 * 2}$$

$x_1 = 0.6385$

$x_2 = -1.3052$

x_2 è da scartare perché l'esponenziale non è mai negativo

$$e^{-0.04t^*} = 0.6385$$

$$-0.04t = \ln(0.6385) = -0.4486$$

$$\longrightarrow t^* = 11.2 \text{ s}$$