Allometry

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Allometric scaling $Y = aM^b$

- Known as the science of scaling in biology
- Has been studied since 1930
- Body parameters are related to mass through power laws

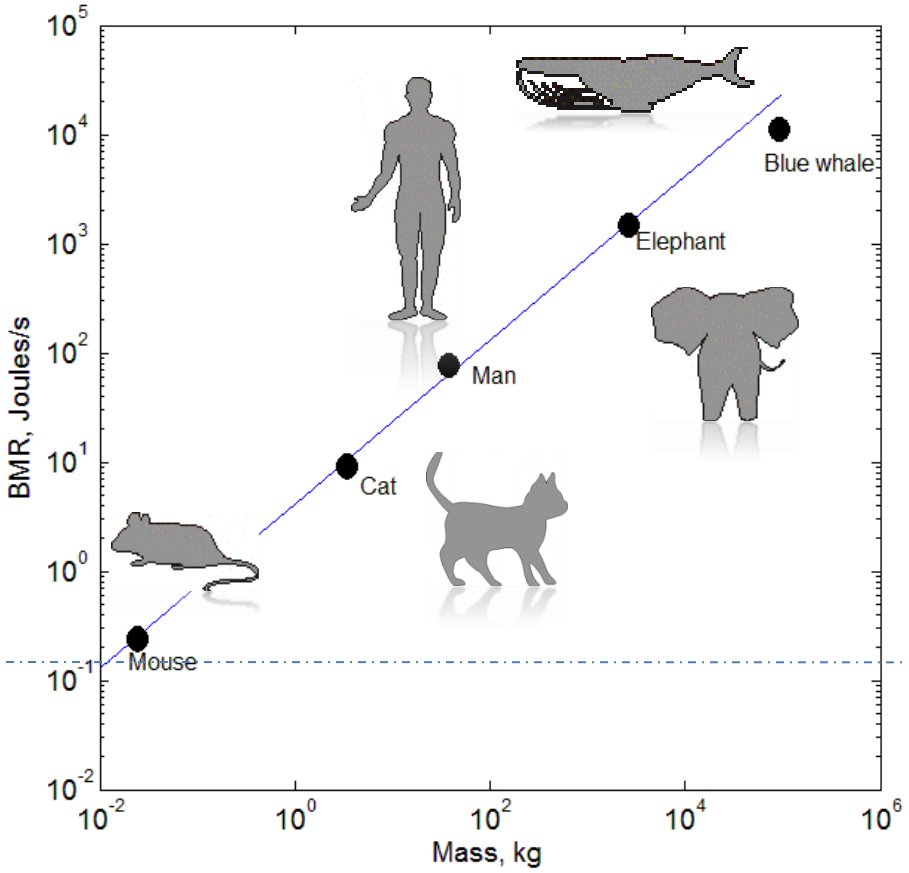
Parameter Y	b
Metabolism	3/4
Cell number	1
Flow rate	1/4

Isometric Scaling

- Proportional scaling, i.e if mass doubles, mass of head, and other organs also doubles. If overall length increases, then all limbs increase proportionately..

BMR / mass relationship, b=3/4

Y

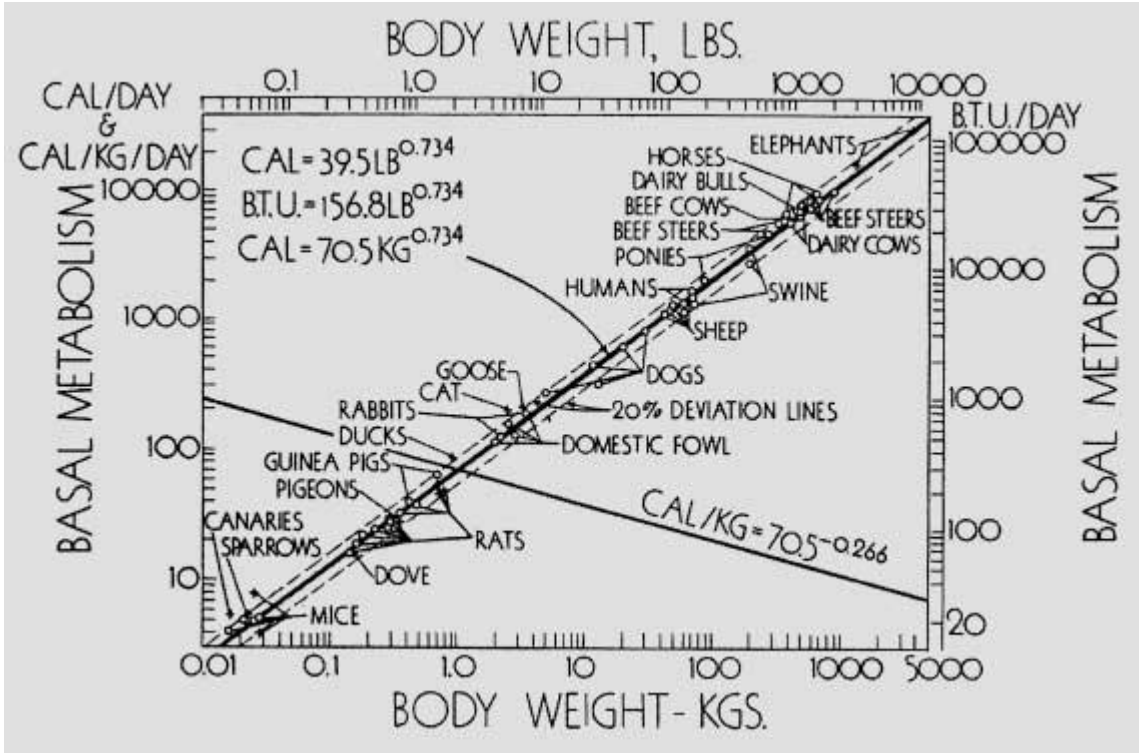


M

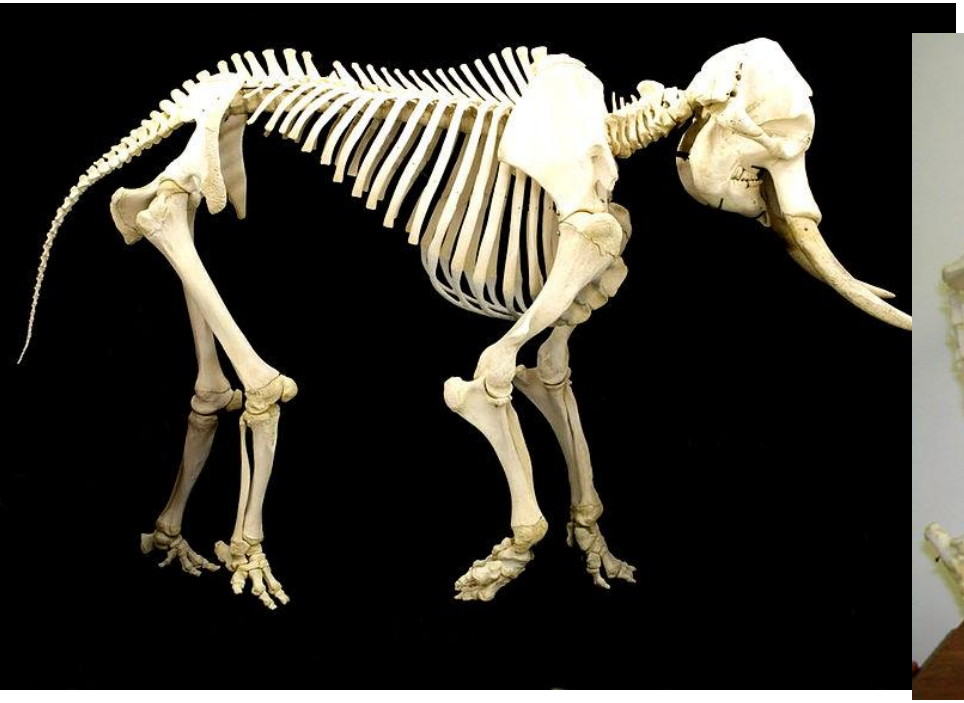
$$Y = aM^b$$

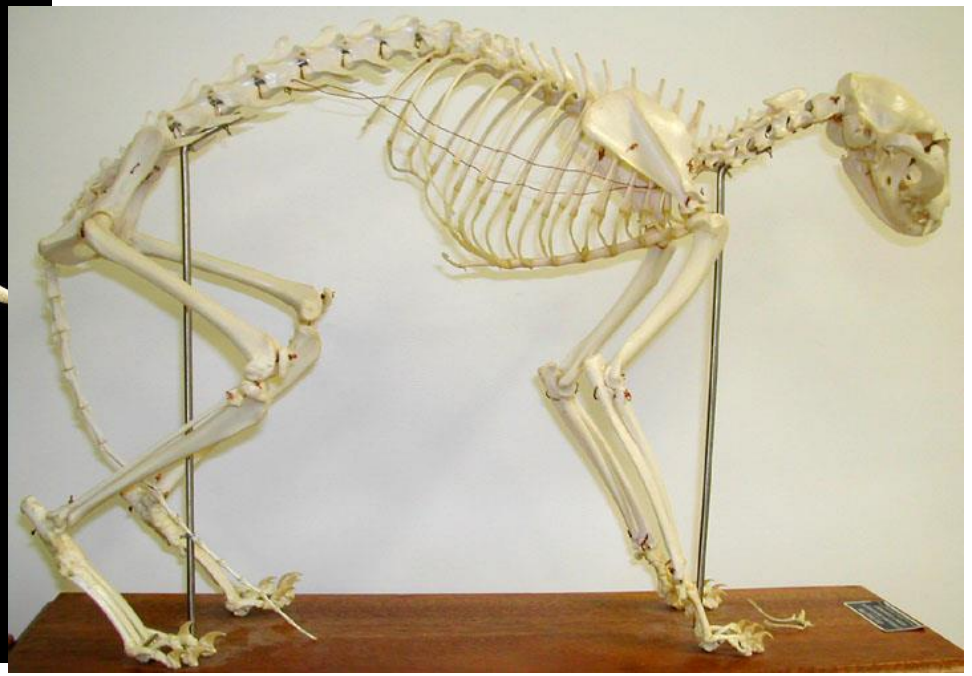
$$\log Y = b \log M + \log a$$

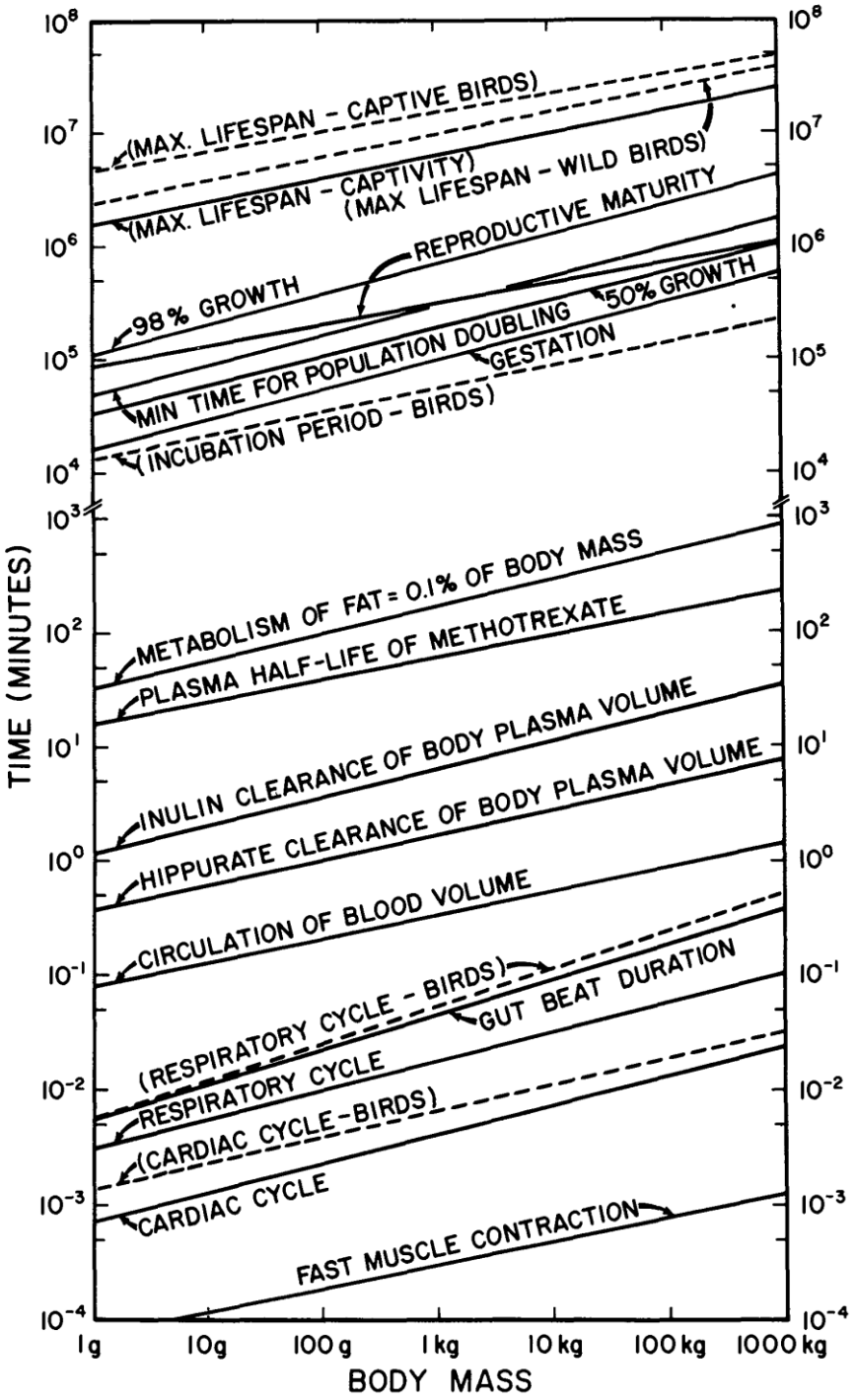
b is the slope
Log a the intercept


BODY WEIGHT, LBS.
0.1
1.0
10
100
1000
10000
CAL/DAY
&
CAL/KG/DAY
B.T.U./DAY
BASAL METABOLISM
BASAL METABOLISM
CAL = 39.5 LB^0.734
B.T.U. = 156.8 LB^0.734
CAL = 70.5 KG^0.734
10000
1000
100
10
100000
10000
1000
100
20
HORSES
ELEPHANTS
DAIRY BULLS
BEEF COWS
BEEF STEERS
PONIES
HUMANS
SWINE
SHEEP
DOGS
GOOSE
CAT
RABBITS
DUCKS
GUINEA PIGS
PIGEONS
CANARIES
SPARROWS
MICE
RATS
DOVE
DOMESTIC FOWL
20% DEVIATION LINES
CAL/KG = 70.5 - 0.266
0.01
0.1
1.0
10
100
1000
5000
BODY WEIGHT - KGS.

Bone mass






TIME (MINUTES)
BODY MASS
(MAX. LIFESPAN - CAPTIVE BIRDS)
(MAX. LIFESPAN - CAPTIVITY)
(MAX LIFESPAN - WILD BIRDS)
98% GROWTH
REPRODUCTIVE MATURITY
MIN TIME FOR POPULATION DOUBLING
50% GROWTH
GESTATION
(INCUBATION PERIOD - BIRDS)
METABOLISM OF FAT = 0.1% OF BODY MASS
PLASMA HALF-LIFE OF METHOTREXATE
INULIN CLEARANCE OF BODY PLASMA VOLUME
HIPPURATE CLEARANCE OF BODY PLASMA VOLUME
CIRCULATION OF BLOOD VOLUME
(RESPIRATORY CYCLE - BIRDS)
GUT BEAT DURATION
RESPIRATORY CYCLE
(CARDIAC CYCLE-BIRDS)
CARDIAC CYCLE
FAST MUSCLE CONTRACTION
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1g
10g
100g
1kg
10kg
100kg
1000kg

$$Y = aM^b$$

b	Significance	Example (*b* value)
0	Parameter does not change with body mass	Bone density in mammals, cell radius
1	Parameter changes in direct proportion with body mass	Body volume, cell number
0<b<1	Parameter increases at a slower rate than body mass	Metabolic rate (3/4), blood flow rate (3/4), external surface area (2/3), life span (1/4)
>1	Parameter increases at a faster rate than body mass	Bone mass (4/3)
<0	Parameter increases with body mass	Almost all frequencies or rates (cardiac frequency, respiratory frequency, -1/4)

Table 1. Values of allometric exponents for variables of the mammalian cardiovascular and respiratory systems predicted by the model compared with empirical observations. Observed values of exponents are taken from (*2*, *3*); ND denotes that no data are available.

Cardiovascular			Respiratory		
Variable	Exponent		Variable	Exponent	
	Predicted	Observed		Predicted	Observed
Aorta radius r_0	3/8 = 0.375	0.36	Tracheal radius	3/8 = 0.375	0.39
Aorta pressure Δp_0	0 = 0.00	0.032	Interpleural pressure	0 = 0.00	0.004
Aorta blood velocity u_0	0 = 0.00	0.07	Air velocity in trachea	0 = 0.00	0.02
Blood volume V_b	1 = 1.00	1.00	Lung volume	1 = 1.00	1.05
Circulation time	1/4 = 0.25	0.25	Volume flow to lung	3/4 = 0.75	0.80
Circulation distance l	1/4 = 0.25	ND	Volume of alveolus V_A	1/4 = 0.25	ND
Cardiac stroke volume	1 = 1.00	1.03	Tidal volume	1 = 1.00	1.041
Cardiac frequency ω	−1/4 = −0.25	−0.25	Respiratory frequency	−1/4 = −0.25	−0.26
Cardiac output $\dot{E}$	3/4 = 0.75	0.74	Power dissipated	3/4 = 0.75	0.78
Number of capillaries N_c	3/4 = 0.75	ND	Number of alveoli N_A	3/4 = 0.75	ND
Service volume radius	1/12 = 0.083	ND	Radius of alveolus r_A	1/12 = 0.083	0.13
Womersley number α	1/4 = 0.25	0.25	Area of alveolus A_A	1/6 = 0.083	ND
Density of capillaries	−1/12 = −0.083	−0.095	Area of lung A_L	11/12 = 0.92	0.95
O_2 affinity of blood P_{50}	−1/12 = −0.083	−0.089	O_2 diffusing capacity	1 = 1.00	0.99
Total resistance Z	−3/4 = −0.75	−0.76	Total resistance	−3/4 = −0.75	−0.70
Metabolic rate B	3/4 = 0.75	0.75	O_2 consumption rate	3/4 = 0.75	0.76