Soft Actuation in Cyclic Motions: Stiffness Profile Optimization for Energy Efficiency

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Abstract— **In this paper, we investigate the role of variable stiffness in the reduction of the energy cost for mechanical systems that perform desired tasks. The objective is to assess the use of Variable Stiffness Actuation (VSA) by determining an optimal stiffness profile and the associated energy cost of performing a desired task. For the analysis we consider mechanical systems of n-Degrees of Freedom (DoF), using VSA. We find an analytical solution that expresses the optimal stiffness profile during the task as a function of joint trajectories. This stiffness profile can be either constant or variable in time, and it minimizes a cost function, when performing a desired task. We calculate the cost related to the torque of the system and the additional cost of changing or keeping a stiffness actively constant. Additionally, we discuss some cases for which it is worth to change the stiffness during a task and cases for which a constant stiffness may be better solution. Furthermore, from simulations and experiments we show cases in which using a variable stiffness profile allows cost savings w.r.t. constant stiffness. The use of variable stiffness depends on the task, i.e. on the joint trajectories and their frequency, as well as on the mechanical implementation of the actuator used.**

I. Introduction

In the last years, compliant actuation has been introduced in robotics mainly to provide safe interaction between humans and machines [1]. Recently, other advantages of soft actuation have been presented; for instance, to have more natural motions or to provide energy efficiency [2], which is relevant in the analysis and design of humanoid robots. According to [3] and [4], much progress has been made in developing robots that can operate in human environments and perform several tasks such as driving a car, opening a valve, walking or removing debris. However, these humanoid robots have been designed aiming to performance and not to efficiency, which limits their functionality [3]. PROXI™[5] is a new humanoid robot platform developed to achieve high efficiency and high performance as required by new robot's generation. For the design, three approaches are considered, namely a transmission that reduces friction, electric motors and batteries and compliance to perform desired gaits.

Based on the importance of energy efficiency for humanoid robotics and given the advantages of soft actuation, this paper focuses on investigating the role of variable stiffness actuation for reducing energy consumption. Cyclic tasks are interesting for our work, because many robotic tasks are or can be modeled as periodic, like walking and hopping, hammering, or the pick and place task. Notice that cyclic may not mean sinusoidal, but a repetitive motion that can be shaped or designed carefully.

Regarding compliant actuators, these mechanisms can have fixed or variable stiffness, depending on their construction. In the first case, the elastic element is constant for all the task and can be placed in series between a high impedance actuator and the load [6], or in parallel to the motor, between the links [7]. In the second case, the stiffness of the actuator can be changed mechanically during a task (VSAs) [8], or either the damping and stiffness can be adjusted (VIAs) [9]. In a previous work [10] we studied the role of soft actuation in the reduction of the energy cost for mechanical systems that perform cyclic motions. We considered the use of Series Elastic Actuators (SEAs) and Parallel Elastic Actuators (PEAs). The optimal stiffness value and spring pre-load were determined such that a given cost functional was minimized. In the mentioned work, the general problem in which both trajectories and actuation parameters were the optimization variables, was cast as a simpler problem in which optimization regarded only trajectories. Although stiffness was considered constant, we discussed that changing it during the task could help to reduce more the energy consumption. Here Variable Stiffness Actuation (VSA) plays an important role. In this paper we tackle the problem of minimizing energy consumption of mechanical systems of n DoF, performing a desired task. The objective is to assess the use of VSA by determining the optimal stiffness profile and the associated energy cost of performing the task. The actual novelty and utility of this method is that the cost function takes into account a weighted cost of the change of stiffness, and the optimization is done during the task.

In literature, different works address this issue from different points of view. Advantages of soft actuation in terms of energy efficiency are tackled e.g. in [11], where the natural dynamics of the mechanism is used to investigate the role of compliance in locomotion. In [12] the energetic cost of leg swinging in dynamic robots is reduced by emulating the use of passive joint stiffness, suggesting as well that similar efficiency improvements could be obtained in dynamic walking robots. Moreover, optimization of stiffness and gait synthesis for biped walking are also addressed e.g. in [13], where simultaneous optimization of the gait and a fixed leg spring stiffness is carried out. On the other hand, periodic tasks are considered e.g. in [14] that proposes an energy efficient method for carrying out a pick and place task. In this approach, an adaptive elastic device with PEAs in each joint is used to reduce the total energy of the task. From a safety point of view, variable stiffness is addressed for instance in [15], where a variable stiffness joint (VSJ) was designed for a robot manipulator. A control scheme of the stiffness and the position of the VSJ are developed. To save energy, [16] presents a compliance controller which continuously changes the stiffness of the actuator to adapt the natural motion of the system to a desired trajectory. Different from our choice, in this case, pneumatic artificial muscles are studied, showing that actuator compliance significantly reduces the energy consumption. In [17] an energy saving control method through stiffness adaptation in PEAs and delayed feedback

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control are simultaneously carried out to generate passive periodic motions. Optimal control strategies to exploit the advantages of VSA have been also studied for example in [18], which addresses a model-based control method to send constrained commands to the VSAs.

In this paper we present a methodology to find a stiffness profile in time, obtaining an analytical solution for optimal stiffness. We discuss some cases for which it is worth to change the stiffness during a desired task and other cases for which a constant stiffness may be a better solution. From the simulations and experiments, we show particular cases in which using a variable stiffness profile leads to obtain savings at least 16.75% w.r.t. constant stiffness in the worst case.

II. Problem Formulation

As mentioned, we are interested in studying soft-robotics actuation, particularly VSA for a n-DoF mechanical system. The purpose is to find the optimal stiffness that minimizes a cost function based on energy consumption when performing a cyclic task. This stiffness can either vary or remain constant during the task. To determine the optimal stiffness profile, we quantify the cost related to the torque and the additional cost of actively changing or keeping the stiffness constant in a VSA mechanism. In this section we state the model of the mechanical systems actuated by VSAs; then we derive the cost related to varying or keeping the stiffness constant. Afterwards, we write the general cost index that we will minimize. At the end, we propose a strategy to find the stiffness profile, considering equal time intervals in order to state the optimization problem.

A. Dynamics of the mechanical system

Consider a mechanical system of n-DoF actuated by VSA whose dynamic equations are described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + K(q-\theta) = 0 \tag{1}$$

$$J_m\ddot{\theta} + K(\theta - q) = \tau\,, \tag{2}$$

where $M(q) \in \Re^{n\times n}$, $C(q,\dot{q}) \in \Re^{n\times n}$, $J_m \in \Re^{n\times n}$ and $K(t) \in \Re^{n\times n}$ are respectively the matrices of inertia, Coriolis, motor's inertia and stiffness; $G(q) \in \Re^n$ is the gravity vector, $q \in \Re^n$, $\dot{q} \in \Re^n$, $\ddot{q} \in \Re^n$ are respectively the vectors of link positions, velocities and accelerations, $\theta \in \Re^n$ and $\ddot{\theta} \in \Re^n$ are the vectors of motor's positions and accelerations, and $\tau \in \Re^n$ is the vector of joint torques.

1) Assumptions: For the analysis, the stiffness and the motor's inertia matrices, $K(t)$ and J_m are diagonal; the link trajectory is given and it is periodic $q(t) = q_d(t+T)$, where T is the period. The stiffness can be actively changed from a minimum value K_{min} to a maximum value K_{max}. We consider that the time required to change the stiffness is negligible w.r.t. other time constants of the systems. Under this assumption it is reasonable to consider the stiffness profile piecewise constant. However, if the stiffness settling time is smaller there may be undesired effects that will be studied in further works.

B. Cost related to stiffness

To assess whether the use of VSA is worth in terms of energy consumption, we take into account the cost of actively varying the stiffness or keeping it constant. We derive the cost of varying the stiffness given a VSA. This cost depends on the stiffness and on the motors torque. In general, the

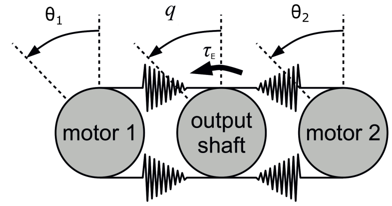


Fig. 1. Agonist Antagonist VSA scheme. τ_E is the torque at the output shaft, θ_1 and θ_2 are the positions of the mechanism's motors, q is the link position

external torque of a VSA can be described by a function $l(\cdot)$

$$\tau_E = l(\tau_1, \tau_2)\,, \tag{3}$$

and the mechanism motors' torques τ_1 and τ_2 can be expressed by general functions $t_1(\cdot)$ and $t_2(\cdot)$

$$\tau_1 = t_1(q, \theta_1, \theta_2)\,, \tag{4}$$

$$\tau_2 = t_2(q, \theta_1, \theta_2)\,. \tag{5}$$

The stiffness is defined as

$$\sigma = \frac{\partial \tau_E}{\partial q}\,. \tag{6}$$

Replacing both (4) and (5), in (3) and in (6) we obtain the external torque and the stiffness respectively, as functions of the link position q and the motors positions θ_1 and θ_2. Now, given the link's reference position $\bar{q}$, the desired external torque $\bar{\tau}_E$ and the stiffness reference $\bar{\sigma}$, we solve the nonlinear system of equations given by (3) and (6), obtaining θ_1 and θ_2. Thus, we can calculate τ_1 and τ_2 from (4) and (5), respectively. Notice that as we are interested in the effect that changing stiffness has in the torque, the desired external torque can be $\bar{\tau}_E = 0$. At this point, we are able to define a function $f^2(\cdot)$ related to the stiffness change as

$$f^2(\cdot) = \tau_1^2 + \tau_2^2\,. \tag{7}$$

Consider that $K_j = \sigma$ is the stiffness of the j-th joint of a n-DoF system, and approximate the related cost of varying the stiffness in the n-DoF mechanical system actuated by VSA as a linear function of the stiffness, so it holds

$$J_K = \sum_{j=1}^{n}\int_0^T \lambda(K_j - K_r)dt\,, \tag{8}$$

where K_r is the stiffness value when no load is applied; $\lambda \in \Re^+$ is a weight that depends on the motor mechanics relating the required torque to keep a certain value of stiffness with the stiffness value. The choice of λ depends on the design of the VSA. We set the minimum σ_{min} and maximum σ_{max} stiffness values to interpolate a linear function between these points such that

$$\lambda = \frac{f^2(\sigma_{max}) - f^2(\sigma_{min})}{\sigma_{max} - \sigma_{min}}\,. \tag{9}$$

Notice that in this way it always holds that $\tau_E^2 < J_K$, because the considered cost $J_K = \lambda(K_j - K_r)$ could overestimate the real cost (see Fig. 2). Furthermore, the weight λ can be calculated several times according to the problem and to the actuators design. Thus, this approach is accurate enough to determine the cost of actively changing

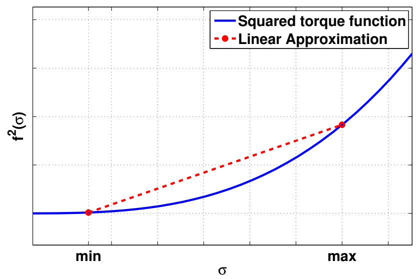


Fig. 2. Comparison of the actual quadratic torque curve with the linear approximation as functions of the stiffness in a VSA model.

or keeping the stiffness constant. This weighted cost is taken into account in the cost index, as presented in the following subsection.

Let us consider an agonist-antagonist VSA as in Fig. 1 (for complete reference on the design, the reader is encouraged to review [19]). This actuator has two motors in opposition, and a set of tensioning mechanisms controlled by servomotors. The motors position are noted as θ_1 and θ_2 and q is the desired link position. As is the case for each antagonistic setup, both actuators are used to influence only one variable, either the compliance or the equilibrium position. According to Fig. 1 the torque at the output shaft is given by

$$\tau_E = \tau_1 + \tau_2 \,, \tag{10}$$

where τ_1, τ_2 are the motors torques. Using the definition of stiffness in (6), σ is given by

$$\sigma = \frac{\partial \tau_1(q-\theta_1)}{\partial q} + \frac{\partial \tau_2(q-\theta_2)}{\partial q} \,.$$

Let the motor torques be written as

$$\tau_1 = a(q-\theta_1)^3 \tag{11}$$
$$\tau_2 = a(q-\theta_2)^3 \,, \tag{12}$$

where $a \in \Re$ is scale factor. Replacing (11) and (12) in (10), it yields $\tau_E = a(q-\theta_1)^3 + a(q-\theta_2)^3$. If the torque at the output shaft is $\bar{\tau}_E = 0$ and the desired link position $\bar{q} = 0$, then we have,

$$a(\theta_1^3 + \theta_2^3) = 0 \,, \qquad \theta_1 = -\theta_2 \,, \qquad \sigma = 6a\theta_1^2 \,.$$

It follows that

$$\theta_1 = \sqrt{\frac{\sigma}{6a}} \,, \qquad \tau_1 = -a\left(\frac{\sigma}{6a}\right)^{3/2} \,, \qquad \tau_2 = a\left(\frac{\sigma}{6a}\right)^{3/2} \,.$$

Then, the function of the motors' torques is

$$f^2(\sigma) = 2a^2\left(\frac{\sigma}{6a}\right)^3 \,. \tag{13}$$

Fig. 2 plots the relation in equation (13) and the approximated linear interpolation.

C. Cost index

To determine the optimal stiffness $\hat{K}$ that minimizes the energy consumption of a n-DoF VSA mechanical system, we assume that its energy consumption is mainly related to the torque, which is proportional to the current spent by the motors. Actually, the energy consumption is a contribution of two terms, one related to the squared current and a term related to the power. The latter is hard to consider because it depends on how the mechanism recovers the power. Minimizing the proposed index is likely to increase energy efficiency. According to [20] a useful measure of the energy efficiency is the cost function based on the squared torque,

$$J_{2,j} = \int_0^T \tau_j^2(t)dt \,. \tag{14}$$

where T represents the period of the cyclic task or a multiple; τ_j is the torque of the j-th joint. Then, considering the cost of actively keeping or varying stiffness, the cost function is expressed as

$$J = \sum_j J_{2,j} + J_{K_j} \,. \tag{15}$$

Define $\psi_{\tau,j} = \tau_j^2 + \lambda(K_j - K_r)$, then, the cost function to minimize for the j-th joint is

$$J_j = J_{2,j} + J_{K_j} = \int_0^T \tau_j^2(t) + \lambda(K_j - K_r)dt = \int_0^T \psi_{\tau,j}dt \,. \tag{16}$$

With these elements we define a strategy to find the optimal stiffness profile as we show in the following.

D. Stiffness profile

The energy consumption of a n-DoF mechanical system that performs a cyclic motion can be minimized by finding the optimal constant stiffness of each joint, that minimizes the cost function J_j in (16). Based on this statement and using a VSA, for certain tasks, the energy consumption can be further reduced. Given $q(t) = q_d(t)$, the cost in (16) for the j-th joint can be written as

$$\begin{aligned} J_j &= \int_0^{T_1} \psi_{\tau,j}dt + \int_{T_1}^{T_2} \psi_{\tau,j}dt + \cdots + \int_{T_{m-1}}^{T_m} \psi_{\tau,j}dt \\ &= \sum_{k=1}^{m} \int_{T_{k-1}}^{T_k} \psi_{\tau,j}dt \,. \end{aligned} \tag{17}$$

Notice that $T = T_m$; each T_k we find the optimal constant stiffness K_k for the jth joint, minimizing the cost function in the k-th interval

$$\min_{K_k} \int_{T_{k-1}}^{T_k} \psi_{\tau,j}dt \,. \tag{18}$$

To solve (18) we propose the methodology presented in the following section.

E. Statement of the Optimization Problem

The optimization problem can now be stated in order to find the optimal stiffness $\hat{K}(t)$ that minimizes the cost function J

$$\min_{K(t),\psi_\tau(t)} J(q,\dot{q},\ddot{q})$$
$$s.t.$$
$$\begin{cases} (1), (2) \\ q(t) = q(t+T) \\ \xi_1(q,\dot{q},\ddot{q}) \leq 0 \\ \xi_2(q,\dot{q},\ddot{q}) = 0 \\ K_m \leq K \leq K_M \end{cases} \tag{19}$$

where K_M, and K_m are the limits of the stiffness during the whole period. Moreover, the nonlinear constraints ξ_1 and ξ_2 define the task. These constraints depend on q, $\dot{q}$ and $\ddot{q}$.

III. Optimal stiffness solution

An analytical solution for the stiffness that minimizes (16) is obtained in this section. In a similar way as in [10] we exploit the dynamic equations of the mechanical system actuated by VSAs, considering the additional cost of actively changing or keeping the stiffness constant, to write the cost functional as a function of the desired joint trajectories $q(t) = q_d(t)$. Then, we obtain an expression of the stiffness that depends on the system dynamics and that solves the problem stated in (19).

From (1), defining $f = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)$ we have,

$$\theta_j = K_j^{-1} f_j(\ddot{q}_d, \dot{q}_d, q_d, t) + q_{d,j}\,, \tag{20}$$

$$\dot{\theta}_j = K_j^{-1} \dot{f}_j(\ddot{q}_d, \dot{q}_d, q_d, t) + \dot{q}_{d,j}\,, \tag{21}$$

$$\ddot{\theta}_j = K_j^{-1} \ddot{f}_j(\ddot{q}_d, \dot{q}_d, q_d, t) + \ddot{q}_{d,j}\,. \tag{22}$$

Replacing (20) and (22) in (2), the j-th motor torque required to track the desired trajectory $q_d(t)$ is

$$\tau_j = J_{m_j}(K_j^{-1}\ddot{f}_j(\ddot{q}_d, \dot{q}_d, q_d, t) + \ddot{q}_{d,j}) + f_j(\ddot{q}_d, \dot{q}_d, q_d, t)\,. \tag{23}$$

After substituting (8) and (23), in (16) with some algebra, we obtain the cost for the j-th joint in the k-th interval

$$J_{j,k} = \frac{F_{j,k}}{K_{j,k}^2} + \frac{G_{j,k}}{K_{j,k}} + H_{j,k} + T_k\lambda(K_{j,k} - K_r)$$

where

$$F_{j,k} = \int_{T_{k-1}}^{T_k} (J_{m_j}\ddot{f}_j)^2 dt\,,\ H_{j,k} = \int_{T_{k-1}}^{T_k} (J_{m_j}\ddot{q}_{d,j} + f_j)^2 dt\,,$$

$$G_{j,k} = \int_{T_{k-1}}^{T_k} 2J_{m_j}\ddot{f}_j(J_{m_j}\ddot{q}_{d,j} + f_j) dt\,,$$

and $K_{j,k}$ is the is stiffness in the k-th interval. To find the optimal stiffness that minimizes the cost function, we solve the equation

$$\frac{\partial(J_{j,k})}{\partial K_{j,k}} = 0\,,$$

which is

$$\frac{\partial(J_{j,k})}{\partial K_{j,k}} = -\frac{2F_{j,k}}{K_{j,k}^3} - \frac{G_{j,k}}{K_{j,k}^2} + \lambda T_k\,,$$

that can be written as

$$-2F_{j,k} - G_{j,k}K_{j,k} + \lambda T K_{j,k}^3 = 0\,.$$

From the problem it holds that

$$F_{j,k} > 0\,,\ K_{j,k} > 0\,,\ \lambda > 0\ \ , T_k > 0\,.$$

So we solve the equation

$$K_{j,k}^3 - a_j K_{j,k} - b_{j,k} = 0 \tag{24}$$

with

$$a_{j,k} = \frac{G_{j,k}}{\lambda T_k}\,,\ b_{j,k} = \frac{2F_{j,k}}{\lambda T_k}\,.$$

Analyzing the discriminant of (24), from the problem it always holds that $27b_{j,k}^2 > 4a_{j,k}^3$, which means that for the k-th interval, (24) has only one real positive solution reported below.

$$\hat{K}_{j,k} = \frac{(2/3)^{1/3} a_{j,k}}{(9b_{j,k} + \sqrt{3(-4a_{j,k}^3 + 27b_{j,k}^2)})^{1/3}} + \frac{(9b_{j,k} + \sqrt{3(-4a_{j,k}^3 + 27b_{j,k}^2)})^{1/3}}{\sqrt[3]{18}} \tag{25}$$

To prove that this value minimizes the cost function, we use the second derivative criteria, it follows that $\frac{\partial^2 J_{j,k}(\hat{K}_{j,k})}{\partial K_{j,k}^2} = \frac{6F_{j,k}}{\hat{K}_{j,k}^4} + \frac{2G_{j,k}}{\hat{K}_{j,k}^3} > 0$. It holds that $F_{j,k} > 0$ and $K_{j,k} > 0$, then $\hat{K}_{j,k}$ is a minimum for (16) if $G_{j,k} \geq 0$. Otherwise, if $G_{j,k} < 0$, $\hat{K}_{j,k} < \frac{3F_{j,k}}{G_{j,k}}$ is a minimum.

The stiffness profile is

$$\hat{K}(t) = \begin{cases} \hat{K}_1 & 0 \leq t < T_1 \\ \hat{K}_2 & T_1 \leq t < T_2 \\ \vdots & \\ \hat{K}_m & T_{m-1} \leq t \leq T_m \end{cases}$$

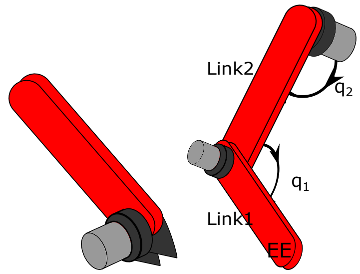
The solution presented is optimal considering a VSA system, but compared to other actuation systems it may not be the best solution in terms of energy consumption. This procedure and the analysis provided aim to give a hint on which actuation systems may be worth to use for a specific task.

IV. Simulation Results

The simulations presented in this section are based on the characteristics of the MakerPro VSA cube [19], whose datasheet is available in [21]. Two case studies are used to analyze the cost of varying stiffness: a One-DoF system and a Two DoF system both actuated by VSA, are simulated performing a pre-defined cyclic motion. For these cases we have considered square trajectories that provide a periodic fast change of position. In the simulations the square trajectories are implemented as

$$q_d(t) = \frac{4}{\pi} A \sum_{n=1,3,5,\ldots} \frac{1}{n} \sin n\omega t \tag{26}$$

because the analytic derivatives are required for determining the optimal stiffness; A is the maximum amplitude of the link, ω is the frequency of the desired cyclic motion, and n is the odd number of harmonics considered to approach the square trajectory; ideally $n \to \infty$, otherwise, a higher frequency ripple may occur; however, this does not affect the cost behavior. For the experiments performed, reported in the next section, we have used an ideal square wave as desired link trajectory and calculated the joint trajectory by inverse kinematics. Physically the system does not respond instantaneously to the change of magnitude in the position, but this has been taken into account in the joint reference calculation. This kind of cycles are used to study particular behavior of the system regarding the cost of energy in cyclic tasks. The tests aim to provide information to analyze in further studies on periodic motions as hammering, running or hopping. By applying the methodology presented we choose either a variable stiffness profile or a constant stiffness for performing a periodic motion based on the cost of changing the stiffness. Here we show that this choice depends strictly



(a) One DoF System (b) Two DoF System

Fig. 3. Mechanical systems considered for the simulation using VSA. EE indicates where an end effector may be placed.

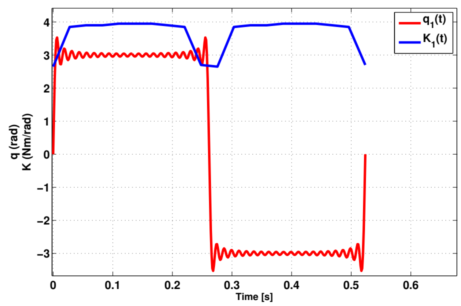


Fig. 4. Stiffness profile $K(t)$ given the desired link trajectory $q(t)$.

on the trajectory, and therefore on the desired task, and we provide some examples.

A. One DoF

Consider a one-DoF system as in Fig. 3(a) actuated by a VSA. The dynamic equations of the system are as written in (1) and (2), and the desired link trajectories for each trial are given by (26). Several simulations have been performed using a set of desired squared trajectories for which we have varied the amplitude and the frequency. For each case we calculate the optimal joint stiffness either constant or variable and we compare the costs. The results suggest that for certain cases it is worth to change the stiffness, i.e. a lower cost can be achieved depending on the reference trajectory, and specifically on the amplitude and the frequency, while in other cases (mainly low amplitudes and low frequencies) it is better to keep the optimal stiffness constant. In Fig. 4 a reference link trajectory and its corresponding stiffness profile are shown. Notice that the change of stiffness is suggested mainly for high frequency changes. Fig. 5 shows the results of the energy cost for a set of desired trajectories whose amplitude is fixed ($A = 3\ rad$) and the frequency varies. Four curves are plotted: one that belongs to the system with a variable stiffness profile and the other three that show the cost of the system using a constant stiffness, chosen as the maximum value considered for the simulations ($K_{max} = 13\ Nm/rad$), the minimum value considered for the simulations ($K_{min} = 0.5 Nm/rad$) and the constant optimal stiffness. The dynamics of the motor and the stiffness parameters are chosen according to the datasheet of the actuator that will be used for the experiment (for complete reference see [8], [21]). In this case, we can see that the cost when using a variable stiffness profile is the lowest.

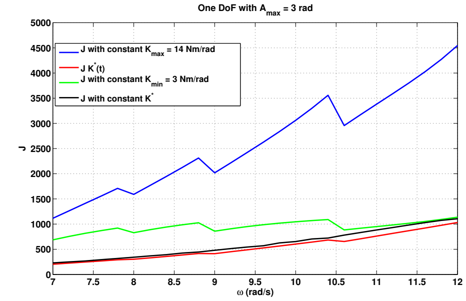
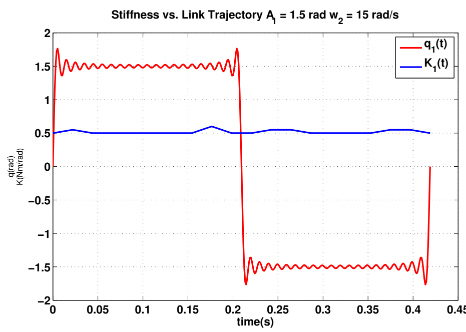
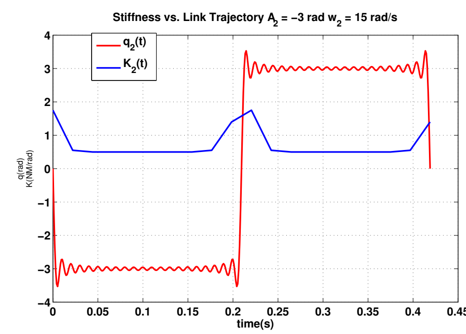


Fig. 5. Energy cost using variable stiffness profile compared to cost using constant stiffness. The plots show the cost variation in 4 cases (constant maximum stiffness; optimal stiffness profile; constant minimum stiffness; constant optimal stiffness), when the frequency changes.



(a) Trajectory $q_1(t)$ and stiffness profile Joint 1



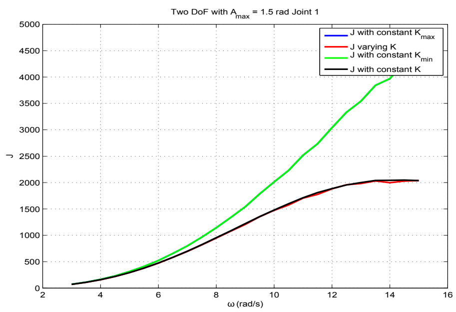
(b) Trajectory $q_2(t)$ and stiffness profile Joint 2

Fig. 6. Optimal stiffness profiles given the desired link trajectories for a Two-DoF system actuated by VSA.

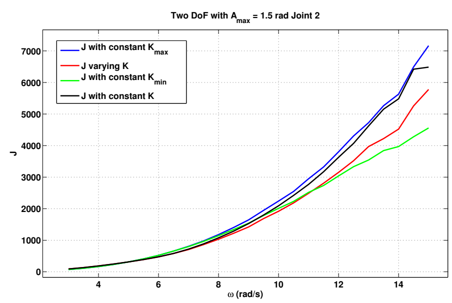
Even though the cost for a constant optimized stiffness is still low and comparable to the cost of the system using variable stiffness, the saving of the latter is at least of 9%.

B. Two DoF

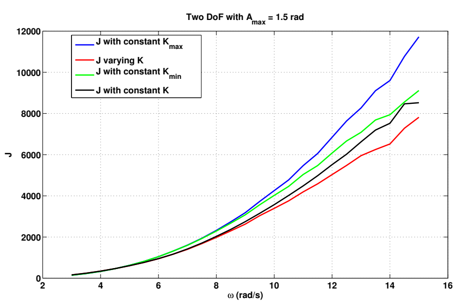
Consider now a two DoF system actuated by VSAs as in Fig. 3(b) whose dynamic equations are (1) and (2). In this case, also a cyclic motion using square trajectories is studied to obtain a linear motion of point EE (refer to Fig. 3(b)). In Figs. 6(a) and 6(b) the desired trajectories to obtain such motion are presented; following the optimization methodology proposed, for each joint we have obtained the shown optimal stiffness. For the Two-DoF case we also present the comparative cost for a set of trajectories of fixed amplitude for which we change the frequency. Figs. 7(a), 7(b) and 7(c) show the cost for Joint 1, Joint 2 and the total cost for the system. For Joint 1 the simulation results show



(a) Cost comparison Joint1



(b) Cost comparison Joint 2



(c) Total Cost comparison

Fig. 7. Cost comparison for a Two-DoF system with constant stiffness (minimum, maximum and optimal) and with a variable stiffness profile

that there is a 0.6% of cost saving when varying the stiffness actively, while for Joint 2 there is a cost saving of 7.4%.

V. Experimental results

A one DoF system based on the qbmove MakerPro VSA[1] [21], as shown in Fig. 8 has been used to carry out a set of experiments that demonstrate the validity of the method proposed. The dynamics of the system is given by (1) and (2), where $M = mL^2 + I$, $m = 0.275\ kg$, $L = 0.098\ m$ $C = 0.001\ Ns/m$.

We perform a set of 390 trials, given a desired joint trajectory $\theta(t)$ that is univocally calculated by inverse dynamics from the link position $q_d(t)$, and after finding the optimal stiffness profile K_j obtained from simulations. The total torque τ_c needed to generate the motion is proportional to the current I_s consumed by the system. It means that $\tau_c = K_t I_s$, where τ_c indicates the total torque needed by the actuators and I_s is the current measured that corresponds to the current consumed by the whole system, including the change of stiffness, and K_t is the electric constant of the motor. The measurements of the current are sampled each $5ms$ and are filtered using a butterworth lowpass filter with cutoff frequency $w_c = 15\ rad/s$. The cost index is calculated as $\int_0^T \tau_c^2 dt$. Notice that the cost of changing the stiffness or keeping it (actively) constant is already considered. We analyze three cases: first, we evaluate the behavior of the system in terms of the energy cost in the case of using the optimal variable stiffness profile obtained from the simulations; second, we obtain the energy cost of the system using the maximum stiffness available (13 Nm/rad); last, we obtain the cost of the system using the minimum constant stiffness available (0.5 Nm/rad). One case of each is plotted in Figs. 9(b), 9(a), and 9(c) we show the desired link and joint trajectories and the corresponding measured trajectories respectively. The chosen case allows to analyze the general behavior of VSA system. Other trials were done changing the desired trajectories. We ensure that the motor follows the desired joint reference appropriately, without affecting the motors effort. On the other hand, the measured link positions are different from the desired link trajectories, due to the system's nonlinearities and non-modeled behaviors. For this study, the idea is to obtain an insight of the behavior of systems using VSA, for which we provide a feedforward strategy, obtaining an optimal stiffness profile. Indeed, for the nature of the system, the position responses are vibrating. A control strategy that combines the optimal solution presented in this paper with a feedback action will be implemented in a future work. However, the system reaches the steady state, and the studied cases are comparable. The costs calculated for each case, are reported in table I. The results obtained indicate that there is a cost reduction of at least 16.75% when using the optimal stiffness profile w.r.t. the worst case.

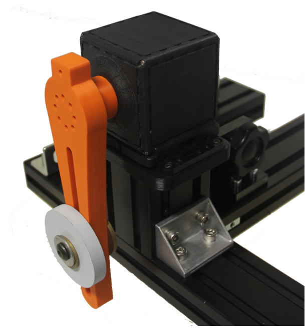


Fig. 8. One DoF platform setup

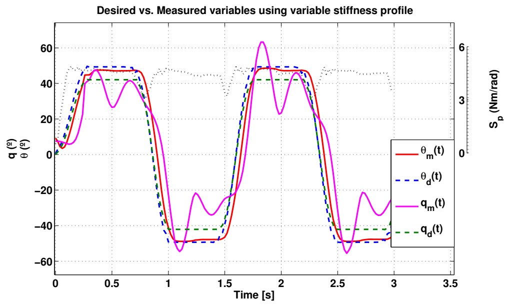
Case	Cost J $[N^2m^2/s]$
Variable stiffness profile	19.16
Maximum stiffness	24.42
Minimum stiffness	22.37

TABLE I
COST CALCULATION FOR EACH CASE STUDY

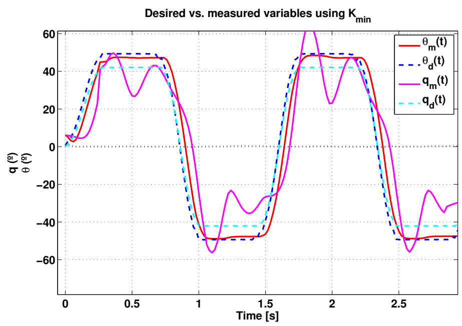
VI. Conclusions

In this paper, we presented a methodology to determine the optimal stiffness temporal profile for the reduction of energy cost for VSA mechanical systems that perform cyclic motions. Simulations were carried out for 1 DoF and 2 DoF models and the experiment is done for the case of 1 DoF,

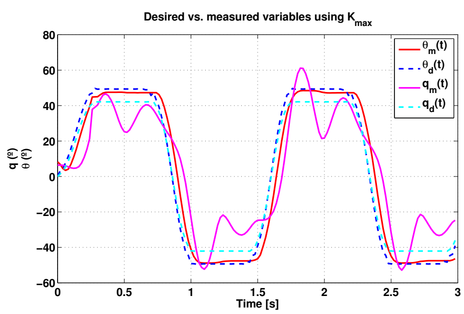
[1]Natural Machine Motion Initiative



(a) Case variable stiffness profile



(b) Case minimum constant stiffness



(c) Case maximum constant stiffness

Fig. 9. Desired Link and motor position vs. Measured positions and measured stiffness preset (in degrees) shown in dashed black line

however it may be applicable the n-DoF case. The latter is left as future study. On the other hand, we have studied cyclic motions because, as introduced before, they are interesting for different robotic applications. This choice may allow also to find the optimal joint trajectories, for example using numerical optimization techniques. However, this issue has already been studied for constant stiffness actuation and can be developed in further studies. The methodology presented in this work can also be applied to noncyclic motions, e.g. point to point tasks, considering a fixed time interval and the corresponding constraints of the motion. Moreover, we found an analytical solution for the optimal stiffness which can be constant or variable, depending on the solution of each sub-period. The advantage of VSA is evident in some of the cases presented for which we show cost savings, but in other cases it may be better to keep the stiffness constant, in terms of energy consumption. Indeed the choice of varying stiffness depends on the task and directly on the trajectories, and on the cost of changing and keeping the stiffness actively constant, due to the mechanical implementation of the VSA.

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