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On the manipulability ellipsoids of underactuated robotic hands with compliance

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ABSTRACT

Underactuation in robotic hands is currently attracting a lot of interest from researchers. The challenging idea of underactuation in grasping is that hands, with reduced number of actuators, supported by suitable design and control, may not suffer from reduced performances. This trend is also strengthened by recent neuroscience studies which demonstrates that also humans use sensorimotor synergies to control the hand in performing grasping tasks. In this paper, we focus on the kinematic and force manipulability analyses of underactuated robotic hands. The performances of such hands, regarded as mechanical transformers of inputs as forces and speed into outputs as object wrench and displacements, are assessed by suitably defined manipulation indices. The whole analysis is not limited by rigid-body motion assumptions, but encompasses elastic motions and statically indeterminate configurations by introducing generalized compliance at contacts and actuation. Two examples show the validity of the proposed approach to evaluate underactuated hand performances.

1. Introduction

Since robotic hands have to usually adapt to many kinds of tasks, they need a complex kinematic structure with a high number of DoFs, which could increase the size, complexity and weight of devices. A possible approach to reduce complexity is that of reducing the number of actuators getting more efficient, simpler and reliable than their fully actuated alternatives [1]. Reducing the number of control inputs, lowers the dimensions of the force and motion controllability subspaces thus affecting the dexterity of the grasp. Studies in the neuroscience demonstrated that a few control variables, named postural synergies, are able to account for most of the variance in the patterns of hand movements and configurations of human hands [2].

Recently, these studies on human hands inspired new research on design and control strategies for robotic hands whose main goal is to achieve a trade-off between simplicity, gained through a synergistic actuation and/or control of DoFs, and its versatility [3,4]. In [4], the synergy idea has been applied to control different hand models: a simple gripper, the Barrett hand, the DLR hand, the Robonaut hand, and the human hand model. In [3] authors proposed a robotic hand design able to match postural synergies mechanically coupling motion of the single joints. In [5] a synergy impedance controller was derived and implemented on the DLR Hand II. In [6] the mapping between the human hand and the robotic hand synergies, with dissimilar kinematics, is proposed and discussed. In [7] the authors investigated to what extent a hand with many DoFs can exploit postural synergies to control force and motion of the grasped object, while [8] analysed how the engaged synergy affect the *quality* of a grasp, in terms of suitably defined cost functions. Different performance measures can be defined to evaluate grasp quality. According to [9] they can be classified in three main groups: the first group considers the position of the contact points and the properties of the grasp matrix [10]. The second group considers the manipulation configuration, for example the Jacobian matrix smallest singular value, or the manipulability ellipsoid volume [11]. The third group of indices takes into account both the kinetic properties of the grasped object and of the manipulators [9].

In this paper we proceed in the analysis of underactuated hands, extending existing manipulability definition. Since their original introduction [12,13], manipulability indices have been widely used in robotics analysis, task specification, and mechanism design. As is well known, the basic idea of manipulability analysis consists of describing directions in the task or joint space that maximize or minimize the ratio between some measure of effort in joint space, and a measure of performance in task space. Whenever these measures are quadratic functions of the joint and task variables, respectively, and the relationships between the two sets of variables is linear, then manipulability analysis amounts to the analysis of an eigenvalue problem. In [14] the kinematic and manipulability analysis was extended to general

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Table 1
Primary notation for grasp analysis.

Notation	Definition
$u \in \Re^{n_d}$	Position and orientation of the object
$w \in \Re^{n_d}$	External wrench applied to the grasped object
n_d	System dimension
n_c	Number of contact points
C_i^o	Reference system at the i-th contact point on the object
$\tilde{c}_i^o \in \Re^{nd}$	Position and orientation of reference frame C_i^o
C_i^h	Reference system at the i-th contact point on the hand
$\tilde{c}_i^h \in \Re^{nd}$	Position and orientation of reference frame C_i^o
$\lambda \in \Re^{n_l}$	Vector of contact forces (and moments)
n_l	Dimension of the contact force vector
n_q	Number of joints
$q \in \Re^{n_q}$	Actual joint variables
$q_r \in \Re^{n_q}$	Reference joint variables
τ	Vector of joint forces and torques
n_z	Number of postural synergies
$z \in \Re^{n_z}$	Synergy variables
$\sigma \in \Re^{n_z}$	Generalized forces along synergies
$G \in \Re^{n_d \times n_l}$	Grasp matrix
$J \in \Re^{n_l \times n_q}$	Hand Jacobian matrix

constrained multibody systems, including parallel robots and multiple cooperative arms in a grasp configuration.

In this work we extended previous results on manipulability analysis to underactuated hands, and particularly to synergy actuated hands. The main aspect here is that in underactuated hands often the force problem cannot be univocally solved within a rigid-body framework, because of static indeterminacy [7,8]. This problem can be solved considering the hand and the contact compliance, as discussed in [15]. In [7,8], a model of underactuated robotic hands is presented which takes into account compliance in the underactuated synergy space. This model is referred to as underactuated hands with soft synergies, this work presents a generalization of the manipulability analysis to such model.

The paper is organized as follows. Section 2 introduces the notation adopted in the paper and presents the quasi-static and kinematic model. Section 3 describes the solution of the quasi-static grasp problem, in particular when the grasp is statically indeterminate. Section 4 extends the manipulability index to synergy actuated hands, and in general to statically indeterminate systems. Section 5 shows two applications of the proposed analysis, the first one to a simple plain gripper, the second one to an anthropomorphic hand.

2. Kinematic and quasistatic model of synergy actuated hands

2.1. Notation and reference systems

In this section the main grasp definitions are summarized, the adopted notation is summarized in Table 1. Consider a robotic hand that grasps an object as in Fig. 1. Let $\{N\}$ represent the inertial frame fixed to the workspace, and let frame $\{B\}$ be fixed to the object. Let n_c be the number of contact points between the object and the robotic hand. Contacts may occur at any place of the robotic hand. At contact point i fixed to the object, the frame $\{C_i^o\}$ is defined with axes $\{\hat{n}_i^o, \hat{t}_i^o, \hat{o}_i^o\}$. The unit vector $\hat{n}_i^o$ is normal to the plane tangent to the contact and directed towards the object. The other two unit orthogonal vectors lie on the tangent plane of the contact. Similarly, we can define the frame $\{C_i^h\}$ with origin at the contact point and fixed to the hand. Let $u \in \mathbb{R}^{n_d}$ denote the vector describing the position and orientation of $\{B\}$ relative to $\{N\}$, where $n_d = 3$ in a two-dimensional (2D) problem, while $n_d = 6$ for a generic three dimensional case.

Vectors $\tilde{c}_i^o \in \Re^{n_d}$ and $\tilde{c}_i^h \in \Re^{n_d}$ describes the position and orientation of the i-th contact reference frame $\{C_i\}$, thought as fixed to the object and to the hand, respectively, with respect to $\{N\}$. Group all these vectors in the overall contact vector $\tilde{c}^o = [\tilde{c}_1^{oT}, \ldots, \tilde{c}_{n_c}^{oT}]^T$ and $\tilde{c}^h = [\tilde{c}_1^{hT}, \ldots, \tilde{c}_{n_c}^{hT}]^T$.

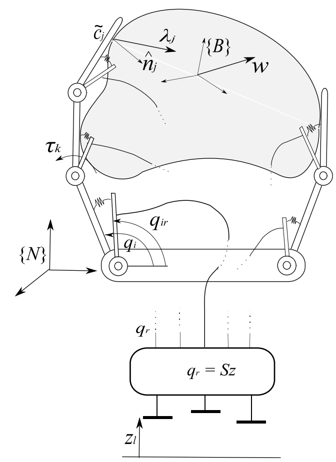

Fig. 1. Hand-object grasp with postural synergies.

In order to define the kinematic constraint and the contact forces imposed by the contact between the hand and the object, a suitable contact model is required. *Single point without friction* (SPWoF), *hard-finger* (HF), and *soft-finger* (SF) [16] are possible examples of a contact model. These realized defining a matrix H that selects a subset of n_l components of the hand and object contact displacements, velocities and contact forces.

The constrained velocities components are coded in the Selection Matrix $H \in \mathbb{R}^{n_l \times n_d n_c}$ [16] which selects the n_l components of the displacement of the contacts: $c^o = H\tilde{c}^o$, $c^h = H\tilde{c}^h$. Let us then define the vectors q_r, and $q \in \mathbb{R}^{n_q}$ of reference and actual joint variables, respectively.

As sketched in Fig. 1, both the contact and the joint servo controller compliance is considered in this work. Let $C_s \in \Re^{n_l \times n_l}$ be the structural compliance matrix, that relates the contact force λ to the displacements of the contact points on the hand and on the object

$$C_s \lambda = (c^h - c^o) \tag{1}$$

and let $C_q \in \Re^{n_q \times n_q}$ be the compliance of the position servo controller, that relates the joint torques $\tau \in \Re^{n_q}$ to the difference between the reference q_r and the actual q joint displacement [15,17]:

$$C_q \tau = q_r - q. \tag{2}$$

The equivalent stiffness matrix K is given by [17]:

$$K = (C_s + JC_q J^{\mathrm{T}})^{-1} \tag{3}$$

where $J \in \Re^{n_l \times n_q}$ is the hand Jacobian matrix and can then be defined by the partial derivatives of the direct kinematic function of the hand with respect to the q components. Since the forward kinematic relationship is generally non linear, matrix J is not constant and depends on joint variables q, disregarding rolling. This relationship is approximated, since it does not take into account the hand Jacobian matrix derivatives. In [18] the validity of this approximation is discussed. In the Appendix a more complete formulation that includes hand Jacobian derivatives in presented.

2.2. Hands controlled with postural synergies

We suppose that the hand is actuated using a number of inputs whose dimension is lower than the number of hand joints. These inputs are referred to as *synergies* and are collected in a vector $z \in \mathbb{R}^{n_z}$.

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This paper refers to postural synergies no matter what type of grasp, human or robotic, is considered.

Differently from other approaches, where the synergistic aggregation of joint displacements is assumed to be rigid, i.e. joint variables are modelled as a linear combination of synergies [19], in this paper we refer to synergies with compliance, i.e. synergies joint displacement aggregations corresponding to a reduced dimension representation of hand movements according to a compliant model of joint torques as described in [7,8].

The *reference* vector q_r for joint variables is a function of *postural synergies* $z \in \mathbb{R}^{n_z}$, with $n_z \leq n_q$. If a *small* variation with respect to a *reference condition* is considered, such a relation can be assumed as linear and the relation between the reference joint vector and the postural synergies can be expressed as

$$\delta q_r = S\delta z, \tag{4}$$

where $S \in \Re^{n_q \times n_z}$ is the *synergy matrix*. Note that the *reference* value of joint rotation q_r may differ from the actual joint displacement q, since a compliant model is assumed [7,8].

2.3. Quasi-static model

Let $w = [f^{\mathrm{T}}\ m_u^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_d}$ be the external wrench (force f, moment m_u) applied to the object. Let $\tau \in \mathbb{R}^{n_q}$ represent joint loads (forces in prismatic joints and torques in revolute joints), $\sigma \in \mathbb{R}^{n_z}$ be the synergy generalized forces, and $\lambda \in \Re^{n_l}$ be the contact force vector.

According to [7,8], the static equilibrium for the grasped object is given by

$$w = -G\lambda \tag{5}$$

where $G \in \Re^{n_d \times n_l}$ is the grasp matrix [16]. In general matrix G is not constant and depends on object displacement, expressed by u, disregarding rolling. The transpose of the grasp matrix relates the displacement of the contact points on the object δc^o to the object reference system displacement δu by the following congruence equation:

$$\delta c^o = G^{\mathrm{T}}\delta u. \tag{6}$$

The contact forces on the hand are balanced by the joint torques $\tau \in \Re^{n_q}$:

$$\tau = J^{\mathrm{T}}\lambda. \tag{7}$$

The Jacobian matrix relates the displacement of the contact points on the hand δc^h to the joint displacement δq by the following congruence equation:

$$\delta c^h = J\delta q. \tag{8}$$

The relation between the joint torques and the synergy forces/torques is given by

$$\sigma = S^{\mathrm{T}}\tau \tag{9}$$

where $\sigma \in \Re^{n_z}$ represents the synergy force/torques. To build the quasistatic model, consider a *variation* with respect to a reference equilibrium condition of the external load $w = w_0 + \delta w$. According to Eqs. (5), (7) and (9), the following equilibrium relationships hold:

$$\delta w + G\delta\lambda = 0 \tag{10}$$

$$\delta\tau - J^{\mathrm{T}}\delta\lambda = 0 \tag{11}$$

$$\delta\sigma - S^{\mathrm{T}}\delta\tau = 0. \tag{12}$$

Note that, to simplify notation, in the main body of the paper we disregard the variations of grasp matrix, hand Jacobian and synergy matrix. The analysis of hand stiffness evaluation including hand Jacobian matrix derivatives is reported in [18]. The above simplification is valid if at least one of the following conditions holds: (1) the system is unloaded or weakly loaded, i.e. if λ_0 is small, (2) if the matrix terms are constant, for example, for the Jacobian matrix, if the hand joints are prismatic, (3) if the joint compliance C_q is small. The quasi-static model, including the derivatives of matrices G, J and S is reported in the Appendix.

The above equilibrium equations can be written in a compact form as

$$\begin{bmatrix} I & 0 & G \\ 0 & I & -(JS)^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \delta w \\ \delta\sigma \\ \delta\lambda \end{bmatrix} = 0. \tag{13}$$

3. Quasi-static problem solution

3.1. Statically determinate system

If and only if the system is not statically indeterminate, i.e. if $\mathcal{N}(G) \cap \mathcal{N}(JS)^{\mathrm{T}} \neq 0$ [16], all the equilibrium combinations of external wrenches, contact forces and synergy generalized forces can be resolved as described in [20,21]:

$$\begin{bmatrix} \delta w \\ \delta\sigma \\ \delta\lambda \end{bmatrix} = \begin{bmatrix} 0 & \Gamma_w & \Gamma_s \\ \Gamma_h & \Gamma_\sigma & 0 \\ \Gamma_{fh} & \Gamma_f & \Gamma_{fs} \end{bmatrix} \begin{bmatrix} x_h \\ x_a \\ x_c \end{bmatrix} \tag{14}$$

where x_h, x_a and x_c parameterize the vector of object wrenches, contact forces and synergy forces satisfying Eqs. (10)–(12).

The first block column of the basis matrix in Eq. (14) corresponds to zero variation of the object wrench. From Eq. (10), it follows that contact forces $\delta\lambda_h = \Gamma_{fh}x_h$ belong to $\mathcal{N}(G)$ and represent the *internal forces*, that do not affect the object equilibrium, and are balanced by the synergy generalized forces $\delta\sigma_h = \Gamma_h x_h$. In a similar way, one can characterize the basis matrix with the third block column in Eq. (14): the contact forces $\delta\lambda_s = \Gamma_{fs}x_s$, that belong to $\mathcal{N}(JS)^{\mathrm{T}}$, represent the *structural contact forces*, balanced by external wrenches $\delta w_s = \Gamma_s x_s$, without the need of synergy forces. Finally, the second block column represents the actual force transmission from the synergy forces $\delta\sigma = \Gamma_\sigma x_a$ to the object wrenches $\delta w = \Gamma_w x_a$, through the contact forces $\delta\lambda\Gamma_f x_a$.

3.2. Statically indeterminate systems

In statics, a structure is statically indeterminate, or *hyperstatic* when the static equilibrium equations are insufficient for determining the internal forces and reactions on that structure.

Considering contact and joint compliance, the contact forces can be expressed as

$$\delta\lambda - K(J\delta q_r - G^{\mathrm{T}}\delta u) = 0 \tag{15}$$

the term $J\delta q_r$ represents the contact point displacement on the hand (evaluated as the joints were perfectly stiff and all the compliance, both at the contact and joint level, were located at the contacts only), while $G^{\mathrm{T}}\delta u$, from Eq. (6), represents contact point displacement on the object, $K \in \Re^{n_l \times n_l}$ is the stiffness matrix, symmetric and positive definite, which includes contact and joint compliance, according to Eq. (3).

Recalling the kinematic relationships between contact points, object and hand displacement, Eqs. (10)–(12) and (15) can be rewritten as

$$\begin{bmatrix} I & 0 & G & 0 & 0 \\ 0 & I & -(JS)^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & I & -KJS & KG^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \delta w \\ \delta\sigma \\ \delta\lambda \\ \delta z \\ \delta u \end{bmatrix} = 0. \tag{16}$$

All possible coordinated forces and motions of the system belong to the nullspace (or kernel) of the constraint matrix on the left-hand term of Eq. (16). All the solutions of Eq. (16) can be written as linear combinations of vectors forming a basis of such nullspace. By suitable algebraic manipulation, such a basis can always be written in a block-partitioned form as (see e.g. [20]):

$$\begin{bmatrix} \delta w \\ \delta\sigma \\ \delta\lambda \\ \delta z \\ \delta u \end{bmatrix} = \begin{bmatrix} 0 & 0 & \Gamma_{w,c} & \Gamma_{w,s} \\ 0 & \Gamma_{\sigma,h} & \Gamma_{\sigma,c} & 0 \\ 0 & \Gamma_{\lambda,h} & \Gamma_{\lambda,c} & \Gamma_{\lambda,s} \\ \Gamma_{z,r} & \Gamma_{z,h} & \Gamma_{z,c} & \Gamma_{z,s} \\ \Gamma_{u,r} & \Gamma_{u,h} & \Gamma_{u,c} & \Gamma_{u,s} \end{bmatrix} \begin{bmatrix} x_r \\ x_h \\ x_c \\ x_s \end{bmatrix}. \tag{17}$$

The first block column of the matrix represents object and hand motions that do not involve contact force variations, i.e. *rigid body motions* [21]. The second block column represents contact force variations (and consequently synergy forces), that correspond to a null variation of the external load δw, thus represents the *internal forces* [15]. The third block column represents external load variations coordinated with contact forces and synergy force variations. Finally the fourth block column represents external load variations balanced by a null variation of synergy forces: these will be referred to as *structural forces* [15].

3.3. Rigid body motion

Rigid-body kinematics are of particular interest in the control of manipulation systems. They do not involve virtual contact spring deformations, and then contact force variations, thus they can be regarded as low-energy motions. Rigid-body kinematics have been studied in a quasi-static setting in [15,22] and in terms of unobservable subspaces from contact forces in [23,24]. In [21] the problem has been analysed also in presence of passive joints. Recalling Eq. (15), and supposing that the internal forces do not change, i.e. $\delta\lambda = 0$, such movements are solutions of the homogeneous system

$$\begin{bmatrix} JS - G^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \delta z \\ \delta u \end{bmatrix} = 0. \tag{18}$$

Then the matrix $[\Gamma_{z,r}^{\mathrm{T}}, \Gamma_{u,r}^{\mathrm{T}}]^{\mathrm{T}}$ in Eq. (17) represents a basis of the nullspace of matrix $[JS \; - G^{\mathrm{T}}]$. As detailed in [21], the basis of all the possible rigid body motions can be further partitioned as

$$\begin{bmatrix} \delta z_{rb} \\ \delta u_{rb} \end{bmatrix} = \begin{bmatrix} \Gamma_{z,rr} & \Gamma_{z,rc} & 0 \\ 0 & \Gamma_{u,rc} & \Gamma_{u,ri} \end{bmatrix} \begin{bmatrix} x_{rr} \\ x_{rc} \\ x_{ri} \end{bmatrix}. \tag{19}$$

In Eq. (19), $\Gamma_{z,rr}$ is a basis matrix of $\mathcal{N}(J)$ and incorporates the *redundancy* of the mechanism: all possible rigid-body motions of the actuated synergies when both the object is locked can be written as linear combinations of columns of $\Gamma_{z,rr}$. Conversely, $\Gamma_{u,ri} = \mathcal{N}(G^T)$ in Eq. (19) represents all possible motions of the object, when actuated synergies are locked. We will refer to the column space of $\Gamma_{u,ri}$ as the *kinematic indeterminacy* subspace of the mechanisms at the given configuration. The second block column of the matrix in Eq. (19) characterizes the coordinated motions of the system. Vectors $\delta u = \Gamma_{u,rc} x_{rc}$ represent the unique possible rigid body motion of the object, coordinated with motions $\delta z = \Gamma_{z,rc} x_{rc}$ of actuated synergies.

4. Manipulability analysis

4.1. Kinematic manipulability

Salisbury and Craig [12], defined the kinematic manipulability index in terms of differential motions, as the ratio of a measure of performance in the task space and a measure of effort in the input (synergies in this case) space:

$$R_k = \frac{\delta u^{\mathrm{T}} W_u \delta u}{\delta z^{\mathrm{T}} W_z \delta z} \tag{20}$$

where $W_u \in \Re^{n_d \times n_d}$ and $W_z \in \Re^{n_z \times n_z}$ are two symmetric and positive definite matrices that weights the different components of δu and δz respectively. R_k can also be interpreted as the ratio between the norm of errors in positioning the end-effector δu, and the norm of errors δz in controlling the synergy actuators to their set-points (the latter errors being regarded as causes of the former). In his original formulation, Yoshikawa [13] defined the kinematic manipulability index in terms of velocities. The analytic formulation of the Yoshikawa's problem and the Salisbury–Craig problem are quite similar, although the results lead to different interpretations.

The analysis of which directions in the task space (and corresponding directions in the actuated joint space) maximize or minimize R_k, is easily solved once a correspondence between the numerator and denominator variables, namely δu and δz, in Eq. (20), is established. Note that, in order for the ratio in Eq. (20) to be well-defined, a one-to-one mapping should be established between the two variables.

From Eq. (17) and (19) it appears that a one-to-one mapping between task and actuated joints velocities does not exist in general, because of the possible presence of redundancy (matrix $\Gamma_{z,rr}$) and indeterminacy (matrix $\Gamma_{u,ri}$). This problem can be circumvented if the physical interpretation of the manipulability ratios is taken into account.

Let us first consider the simpler case in which the grasp is not redundant neither indeterminate. According to Eq. (17) the synergy variation and the object displacement can be expressed as

$$\delta z = \Gamma_{z,rc} x_{rc} + \Gamma_{z,h} x_h + \Gamma_{z,c} x_c + \Gamma_{z,s} x_s = \Gamma_{z,k} x_k \tag{21}$$

$$\delta u = \Gamma_{u,rc} x_{rc} + \Gamma_{u,h} x_h + \Gamma_{u,c} x_c + \Gamma_{u,s} x_s = \Gamma_{u,k} x_k. \tag{22}$$

In this formulation, the object motion can be produced by a coordinated hand/object rigid body motion $\Gamma_{z,rc} x_{rc}$, a variation of the internal forces $\Gamma_{u,h} x_h$, a variation of the structural contact forces $\Gamma_{u,s} x_s$, a coordinated force/displacement variation $\Gamma_{u,c} x_c$. The last three terms are not present when only the rigid-body kinematic analysis is performed, as for example in [20,21]. The compliance model introduces object motions that cannot be analysed with a purely kinematic approach, and these motions causes variations in the system forces. In the more general compliant model, the kinematic manipulability index is then given by

$$R_k = \frac{x_k^{\mathrm{T}} \Gamma_{u,k}^{\mathrm{T}} W_u \Gamma_{u,k} x_k}{x_k^{\mathrm{T}} \Gamma_{z,k}^{\mathrm{T}} W_z \Gamma_{z,k} x_k} \tag{23}$$

where $x_k = [x_{rc}^{\mathrm{T}} \; x_h^{\mathrm{T}} \; x_c^{\mathrm{T}} \; x_s^{\mathrm{T}}]^{\mathrm{T}}$, $\Gamma_{u,k} = [\Gamma_{u,rc} \; \Gamma_{u,h} \; \Gamma_{u.c} \; \Gamma_{u,s}]$, $\Gamma_{z,k} = [\Gamma_{z,rc} \; \Gamma_{z,h} \; \Gamma_{z.c} \; \Gamma_{z,s}]$. As outlined in [20], the maximum value of the Rayleigh quotient corresponds to the maximum eigenvalue of the pencil $\Gamma_{u,k}^{\mathrm{T}} W_u \Gamma_{u,k} - \alpha \Gamma_{z,k}^{\mathrm{T}} W_z \Gamma_{z,k}$. Accordingly, the direction in the space of the parameter x_k where the maximum efficiency is obtained, $x_{k,\max}$ is the generalized eigenvector corresponding to the maximum eigenvalue $\alpha_{\max}$. The corresponding optimal directions in the task and synergy domains are given by $\delta u_{\max} = \Gamma_{u,k} x_{k,\max}$, $\delta z_{\max} = \Gamma_{z,k} x_{k,\max}$.

If the system is redundant and/or indeterminate the synergy variation and object displacement are given by

$$\delta z = \Gamma_{z,k} x_k + \Gamma_{z,rr} x_{rr} \qquad \delta u = \Gamma_{u,k} x_k + \Gamma_{u,ri} x_{ri}. \tag{24}$$

It is thus evident that a one-to-one relationship between task and actuated joint velocities does not exist in general.

If there is redundancy, but no indeterminacy ($\Gamma_{z,rr} \neq 0$), from the Salisbury–Craig viewpoint, redundancy of actuation should conservatively be taken into account as if playing against the mechanism accuracy, hence by considering the *worst-case* controller error δz which, among those compatible with a given δu, minimizes the denominator. On the other hand, if the system is kinematically indeterminate ($\Gamma_{u,ri} \neq 0$), because of the existence of non-zero task frame twists corresponding to zero active joint velocities, the efficiency index in Eq. (20) results unbounded. From the Salisbury–Craig viewpoint, this means that near-singular configurations are very inaccurate. The kinematic manipulability ratio for kinematically redundant and indeterminate mechanisms, which is *optimized* w.r.t. redundancy and *worst-case* w.r.t. indeterminacy, can thus be defined as

$$R_k = \frac{\min\limits_{x_{ri}} x_k^{\mathrm{T}} \Gamma_{u,k}^{\mathrm{T}} W_u \Gamma_{u,k} x_k}{\min\limits_{x_{rr}} x_k^{\mathrm{T}} \Gamma_{z,k}^{\mathrm{T}} W_z \Gamma_{z,k} x_k}. \tag{25}$$

The constrained minimization problem appearing in the numerator and denominator of Eq. (25) can be readily solved by standard linear algebraic tools, as described in [21].

4.2. Force manipulability

The force manipulability index is similarly defined as the ratio of a performance measure in the space of forces exchanged with the environment, and an effort measure in the space of actuated joint torques

$$R_{af} = \frac{\delta w^{\mathrm{T}} W_w \delta w}{\delta \sigma^{\mathrm{T}} W_\sigma \delta \sigma}. \tag{26}$$

Here, weights in $W_\sigma \in \Re^{n_z \times n_z}$ incorporate different costs in generating synergy torque or forces, and take care of mismatches of units of measurement between revolute and prismatic joints. Similarly, weights in $W_w \in \Re^{n_d \times n_d}$ take care of different inhomogeneous physical quantities in the n_d-dimensional wrench δw, and may represent task specifications (such as greater leverage in some direction). A physically motivated choice could be taking W_w as the stiffness matrix of the environment with which the reference member interacts. In this case, the numerator of Eq. (26) would represent twice the elastic energy of interaction.

4.2.1. Active force manipulability

For a given set of equilibrium torques at the actuated synergies, i.e., for given x_h and x_c, in Eq. (17), the corresponding wrench is not uniquely defined if a nullspace of $(JS)^{\mathrm{T}}$ exists (structural forces). However, in the worst case, when wrench variations δw are considered to play against maximization of the index, efficiency will be given by

$$R_{af}^{w} = \frac{\min\limits_{x_s} \delta w^{\mathrm{T}} W_w \delta w}{\delta \sigma^{\mathrm{T}} W_\sigma \delta \sigma}. \tag{27}$$

Note that, if W_w takes into account the environmental stiffness, minimization of the numerator amounts to assuming that the mechanism applies, for the given synergy forces, the wrench that minimizes the energy of elastic deformation.

Define $x_a = [x_h^{\mathrm{T}}\ x_c^{\mathrm{T}}]^{\mathrm{T}}$, $\Gamma_{w,a} = [0\ \Gamma_{w,c}]$, $\Gamma_{\sigma,a} = [\Gamma_{\sigma,h}\ \Gamma_{\sigma,c}]$, Eq. (27) can be expressed as

$$R_{af}^{w} = \frac{\min\limits_{x_s} x_a^{\mathrm{T}} \Gamma_{w,a}^{\mathrm{T}} W_w \Gamma_{w,a} x_a}{x_a^{\mathrm{T}} \Gamma_{\sigma,a}^{\mathrm{T}} W_\sigma \Gamma_{\sigma,a} x_a}. \tag{28}$$

That can be solved using suitable projector matrices [21]. Also in this case the problem is solved as a generalized eigenvalue problem. The discussion of the ellipsoid is similar to the one given above for kinematic manipulability.

4.2.2. Passive force manipulability

For a given equilibrium wrench acting externally on the reference member, i.e., for given x_c and x_s in Eq. (17), the corresponding synergy forces are not uniquely defined if a nullspace of G (internal contact forces) exists. However, it is reasonable to assume that the controller policy will specify that the torque with minimum cost be chosen to oppose a given wrench. The optimized passive force efficiency will hence be given by

$$R_{pf}^{o} = \frac{\delta w^{\mathrm{T}} W_w \delta w}{\min\limits_{x_h} \delta \sigma^{T} W_\sigma \delta \sigma}. \tag{29}$$

Let us define $x_p = [x_c^{\mathrm{T}}, x_s^{\mathrm{T}}]^{\mathrm{T}}$, $\Gamma_{w,p} = [\Gamma_{w,c}, \Gamma_{w,s}]$, $\Gamma_{\sigma,p} = [\Gamma_{\sigma,c}, 0]$, the optimized passive force efficiency can be expressed as

$$R_{pf}^{o} = \frac{x_p^{\mathrm{T}} \Gamma_{w,p}^{\mathrm{T}} W_w \Gamma_{w,p} x_p}{\min\limits_{x_h} x_p^{\mathrm{T}} \Gamma_{\sigma,p}^{\mathrm{T}} W_\sigma \Gamma_{\sigma,p} x_p}. \tag{30}$$

Again, the problem can be solved using the projector notation described in [21].

5. Numerical experiments

5.1. Simple gripper

Let us consider the example shown in Fig. 2, in which a two finger hand is grasping a square object. Each finger consists of three links and three joints. The grasp has four contact points: $C_1, \ldots, C_4$ and a hard finger (HF) model is assumed to model the contact interaction. The contact point coordinates, expressed with respect to the XY reference system shown in the figure, are: $C_1 = [-2a, a]^{\mathrm{T}}$, $C_2 = [-2a, 5a]^{\mathrm{T}}$, $C_3 = [2a, 5a]^{\mathrm{T}}$, $C_4 = [2a, a]^{\mathrm{T}}$. The joint coordinates, in the reference configuration analysed in this section, are: $J_1 = [-a, 0]^{\mathrm{T}}$, $J_2 = [-2a, 2a]^{\mathrm{T}}$, $J_3 = [-2a, 4a]^{\mathrm{T}}$, $J_4 = [a, 0]^{\mathrm{T}}$, $J_5 = [2a, 2a]^{\mathrm{T}}$, $J_6 = [2a, 4a]^{\mathrm{T}}$. The joint variable vector is given by $q = [q_1, \ldots, q_6]^{\mathrm{T}}$. The generic variable q_i represents the rotation angle of the joint J_i, evaluated with respect to the adjoining link. In the considered reference configuration, the joint angles are: $q_0 = [3/4\pi, -\pi/4, -\pi/4, \pi/4, \pi/4, \pi/4]^{\mathrm{T}}$.

Grasp matrix, and hand Jacobian, in the reference configuration, are given by [16,25]:

$$G^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 2a \\ 0 & 1 & -2a \\ 1 & 0 & -2a \\ 0 & 1 & -2a \\ 1 & 0 & -2a \\ 0 & 1 & 2a \\ 1 & 0 & 2a \\ 0 & 1 & 2a \end{bmatrix}$$

$$J = \begin{bmatrix} -a & 0 & 0 & 0 & 0 & 0 \\ -a & 0 & 0 & 0 & 0 & 0 \\ -5a & -3a & -a & 0 & 0 & 0 \\ -a & a & a & 0 & 0 & 0 \\ 0 & 0 & 0 & -5a & -3a & -a \\ 0 & 0 & 0 & a & -a & -a \\ 0 & 0 & 0 & -a & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 \end{bmatrix}.$$

In this simple example, we suppose that the hand is underactuated using only two actuators (synergies). The first synergy moves the joint J_1, keeping locked the others, while the second one move

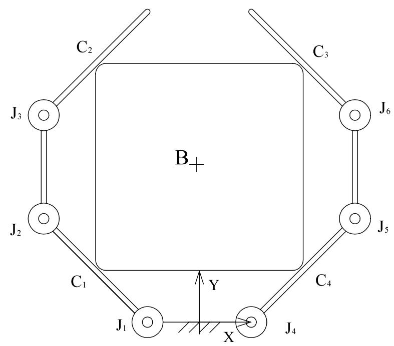

Fig. 2. Robotic hand with two fingers manipulating a square object.

the joint J_4, keeping locked the others. In other terms, in the synergy actuated hand each finger works as a one degree of freedom link. The corresponding synergy matrix is given by

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

For the compliance model we initially assumed that $K = kI$, with k varying from 1 to 10 N/m. Then we performed some tests considering $C_s = I$ and increasing C_q = elements from 0.01 to 1 m/N, and evaluating the stiffness matrix as described in Eq. (3). Finally we considered $C_q = I$ and decreased C_s elements from 1 to 0.01 m/N.

The system in Eq. (16) can be defined and the matrix $\Gamma \in \Re^{18\times 5}$ can be computed. With an algorithm as those described in [20], the matrix can be partitioned as shown in Eq. (17). In this example, the grasp configuration is such that no rigid motion is possible: $\Gamma_{z,r} = \Gamma_{u,r} = 0$. Furthermore, it results that $n_h = 2$ and $n_c = 3$.

It is worth underlining that the absence of rigid body motions means that it is not possible to move the object and the hand without changing the contact forces. In the kinematic manipulability analysis, the object movements corresponding to a null variation of the external load δw were considered: $\Gamma_{u,k} = \Gamma_{u,h}$ and $\Gamma_{z,k} = \Gamma_{z,h}$. Furthermore we considered the translation motion of a specific point, the *tool tip*, corresponding to the centre of the object B.

The kinematic manipulability analysis, obtained assuming $K = kI$ and $k = 1$ N/m, gives the following principal directions in the x_k space:

$$x_{k\min} = [-1.15 \quad -0.04]^{\mathrm{T}} \qquad x_{k\max} = [-0.04 \quad 1.15]^{\mathrm{T}}.$$

These correspond to the following object displacements

$$\delta u_{k\min} = [0.00 \quad -0.07]^{\mathrm{T}} \qquad \delta u_{k\max} = [-0.21 \quad 0]^{\mathrm{T}}$$

and call for the following synergistic variations

$$\delta z_{k\min} = [0.71 \quad -0.71]^{\mathrm{T}} \qquad \delta z_{k\max} = [0.71 \quad 0.71]^{\mathrm{T}}.$$

The obtained results are shown in Fig. 3(a): we observe that the component with the maximum absolute value in $\delta u_{\max}$ is in the X direction and is obtained activating the synergies in the same directions (blue arrows). On the other hand, the direction corresponding to the minimum eigenvalue, $\delta u_{\min}$, is along the Y axis and is obtained activating the synergies with opposite directions (red arrows).

The active force manipulability analysis gives the following principal directions in the x_a space:

$$x_{a,\min} = [-0.13 \quad 0 \quad 1]^{\mathrm{T}} \qquad x_{a,\max} = [0 \quad 1 \quad 0]^{\mathrm{T}}$$

$$x_{a,\infty} = [0.86 \quad 0 \quad 1]^{\mathrm{T}}$$

whose corresponding directions in the δw space are

$$\delta w_{a,\min} = [-0.11 \quad 0 \quad 0.29]^{\mathrm{T}}$$

$$\delta w_{a,\max} = [0 \quad 0.87 \quad 0]^{\mathrm{T}}$$

$$\delta w_{a,\infty} = [0.74 \quad 0 \quad 0.29]^{\mathrm{T}}$$

and the corresponding synergy forces are

$$\delta \sigma_{a,\min} = [-0.13 \quad -0.13]^{\mathrm{T}}$$

$$\delta \sigma_{a,\max} = [-0.04 \quad -0.04]^{\mathrm{T}}$$

$$\delta \sigma_{a,\infty} = [0 \quad 0]^{\mathrm{T}}.$$

The obtained results are shown in Fig. 3 (b): we observe that the object wrench corresponding to the maximum eigenvalue $\delta w_{\max}$ is in the Y direction and is obtained activating the synergies in opposite directions (red arrows). On the other hand, the direction corresponding to the minimum eigenvalue, $\delta w_{\min}$, has a component along the X axis and a torque component (that was not represented in the figure), and is obtained activating the synergies with the same directions (blue arrows).

Table 2 shows the sensitivity of the obtained results, in terms of kinematic ellipsoid semi axes, on the stiffness values. The first three columns are obtained assuming $K = kI$ and varying k from 1 to 10 N/m. As it can be seen, the ellipsoid semi axes do not change varying the stiffness value, in other terms, the kinematic ellipsoid shape and dimension does not depend on k value (of course the corresponding $\delta\lambda$ values change varying the k values). The following columns show the results obtained changing C_q and C_s and evaluating matrix K as shown in Eq. (3). In this case, the shape of the kinematic manipulability ellipsoids change: the effect is much more evident as C_q become relevant with respect to C_s.

5.2. The human hand kinematic model

As a second example we consider the model of an anthropomorphic robotic hand, whose kinematic parameters are described in [8]. With respect to the model presented in [8], here the model has been enriched adding the distal interphalangeal joints for the index, middle, ring and pinkie fingers, and the abduction/adduction degree of freedom of the middle finger metacarpal joint. The resulting kinematic model has 20 degrees of freedom.

The synergy matrix S was calculated using the measurements and the approach based on the principal component analysis proposed in [2].

The reference configurations, described by the joint vector q_m, corresponds to the grasping of different objects: in this section the results corresponding to three of the available objects are presented: an *egg*, a *calculator*, a *pair of scissors*. In Fig. 4 the hand reference posture corresponding to the grasp of an egg is shown. For convenience in the figure the grasped object was represented as a sphere. The joint angles were obtained from the measured data in [2]. Fig. 5 shows the shape of the first three synergies, evaluated starting from the reference configuration and activating the first, the second and the third synergy only.

Once we selected the reference configuration, corresponding to a specific grasped object, the contact point locations were set at the tip of each finger, thus five contact points were defined for each grasp. The Hard Finger contact model was adopted to model the contact between the fingers and the object.

For the given hand configuration, contact points and types, the grasp matrix G and the hand Jacobian matrix J can be calculated as explained in the preceding section and as detailed for example in [16]. Since in this example $n_l = 15$, $n_d = 6$, $n_q = 20$, then $G \in \Re^{6\times 15}$ and $J \in \Re^{15\times 20}$.

To consider the synergistic actuation with the first three synergies, matrix J is then substituted by JS, where $S \in \Re^{20\times 3}$

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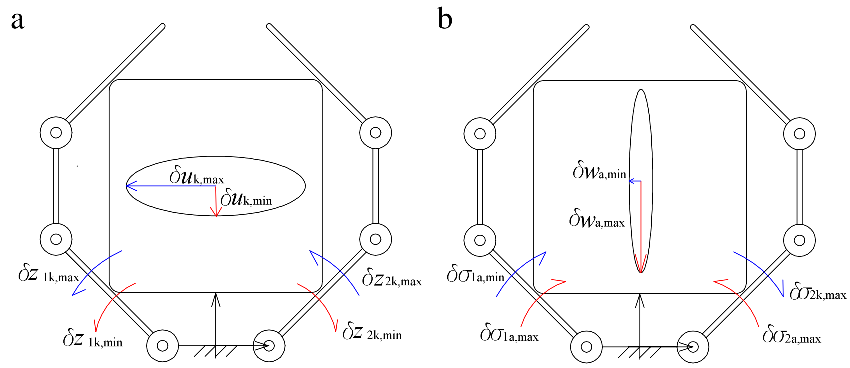

Fig. 3. (a) Kinematic ellipsoids for the two fingered, synergy actuated, gripper. Blue arrows: point B displacement $\delta u_{\max}$ and synergy actuation $\delta z_{\max}$ corresponding to the maximum eigenvalue. Red arrows: point B displacement $\delta u_{\min}$ and synergy actuation $\delta z_{\min}$ corresponding to the minimum eigenvalue. (b) Force ellipsoids. Red arrows: object wrench $\delta w_{\max}$ and synergy force $\delta\sigma_{\max}$ corresponding to the maximum eigenvalue (only the force components δw_1 and δw_2, the torque component δw_3 is not represented). Blue arrows: object wrench $\delta w_{\min}$ and synergy force $\delta\sigma_{\min}$ corresponding to the minimum eigenvalue. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2
Kinematic manipulability analysis on the simple gripper: sensitivity on stiffness values. First three columns: $K = kI_{8\times8}$ with $k = 1, 5, 10$ N/m: Columns 4–6: K evaluated by (Eq. (3)), assuming $C_q = c_q I_{6\times6}$, $C_s = c_s I_{8\times8}$, $c_q = 0.001, 0.1, 1$ m/N, $c_s = 1$ m/N. Columns 7–8: K evaluated by (Eq. (3)), assuming $C_q = c_q I_{6\times6}$, $C_s = c_s I_{8\times8}$, $c_q = 1$ m/N, $c_s = 0.1, 0.01$ m/N.

	$k = 1$	$k = 5$	$k = 10$	$c_q = 0.01$ $c_s = 1$	$c_q = 0.1$ $c_s = 1$	$c_q = 1$ $c_s = 1$	$c_s = 0.1$ $c_q = 1$	$c_s = 0.01$ $c_q = 1$
$x_{k,\min}$	−1.15 −0.04	−3.01 −0.28	−5.75 0.84	−1.15 −0.04	1.14 0.04	−1.09 0.05	−1.64 0.23	−3.04 0.75
$x_{k\max}$	−0.04 1.15	−0.11 1.25	0.22 1.51	−0.04 1.15	−0.05 1.15	−0.05 1.14	−0.18 1.29	−0.7809 3.1639
$\delta u_{\min}$	0 −0.07	0 −0.07	0 −0.07	0 −0.07	0 −0.07	0 −0.0623	0 −0.04	0 0.0218
$\delta u_{\max}$	−0.21 0	−0.21 0	−0.21 0	−0.21 0	−0.21 0	−0.21 0	0.18 0	0.1377 0

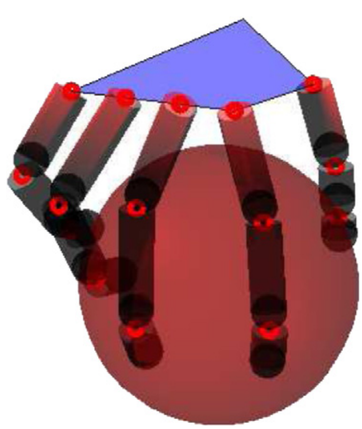

Fig. 4. The robotic hand grasping an egg (represented as a sphere) in the reference configuration.

represents the synergy matrix obtained choosing the first three principal components obtained performing the PCA on hand measures. In the presented results, if $n_z = 3$, then $JS \in \Re^{15\times3}$.

In the synergy actuated case, the kinematic and force manipulability index is analysed. Matrix $\Gamma \in \Re^{33\times9}$, whose columns span such subspace, was evaluated and partitioned according to Eq. (17), using the algorithm described in [20]. This matrix arrangement allows highlighting the contribution of rigid body motion, internal forces, structural forces, coordinated force/motion/synergy variations. In particular, we obtained $n_r = 0$ (no rigid motion are allowed), $n_h = 3$ (the internal force subspace dimension), and $n_c + n_s = 6$. Even if also in this case no rigid body motions are allowed, due to the system compliance, the object displacement is modified when the forces acting on the object, the contact forces, and the synergy forces are varied. In this example, we analysed the kinematic manipulability index defined in Eq. (23), considering not all the possible object motions, but only the solutions that do not imply a variation in the external load δw, in other terms, we considered $x_k = x_h \in \Re^3$. The manipulability analysis, i.e. the study of the Rayleigh ratio in Eq. (23) leads to three eigenvalues, and consequently three eigenvectors: directions in the x_k space. The ratio in Eq. (23) describes in the x_k space an ellipsoid. The displacement δu corresponding to the maximum eigenvalue, for the *egg* grasp, is $\delta u_{\max} = \Gamma_{u,k} x_{k,\max} = [0.01, 0.03, 0.06]^{\mathrm{T}}$ (only the translational part of the object motion is considered). The second column of Fig. 6 shows the kinematic ellipsoids in the δu space for different grasped objects. In all the presented cases, the ellipsoids presents a principal direction along with the efficiency is higher (one of the three semiaxis is sensibly higher than the others). The synergy variation that corresponds to the maximum eigenvalue, for the first case is $\delta z = \Gamma_{z,k} x_{k,\max} = [0.12, 0.95, 0.28]^{\mathrm{T}}$. Similarly, the minimum efficiency direction $x_{\min}$, and the corresponding object displacement $\delta u_{\min}$ and synergy variation $\delta z_{\min}$ can be evaluated. A similar analysis was be performed for the force manipulability ellipsoids, by analysing the ratio defined in Eq. (26), the results, obtained for different objects, is shown in the second column of Fig. 6.

6. Conclusions

Manipulability analysis is commonly used in robotics to measure the performance of a robotic system, expressed as the ratio between a measure of force/velocity in the task space and the corresponding effort in the input actuation system. The manipulability analysis allows to identify the directions, in the input and output space, that maximize and minimize this

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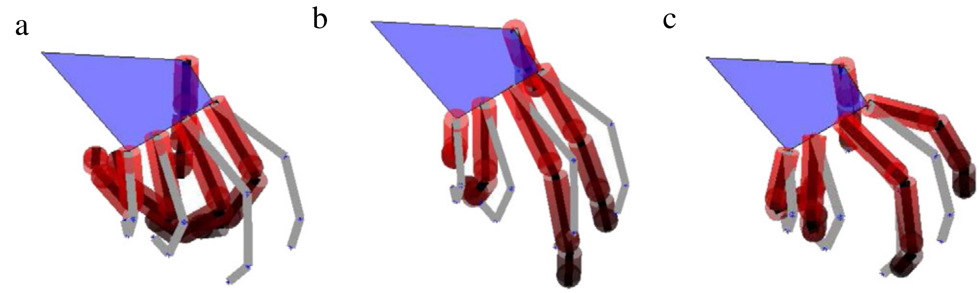

Fig. 5. Hand motion due to the first (a), the second (b) and the third (c) synergy only.

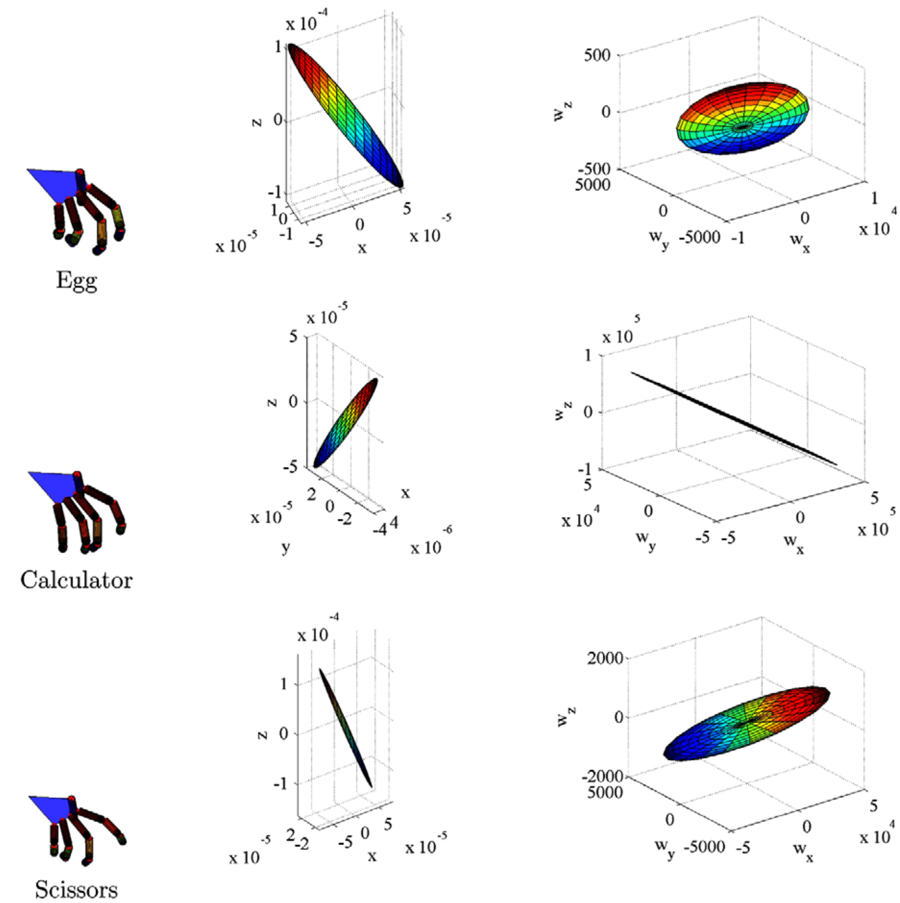

Fig. 6. Manipulability ellipsoids for the anthropomorphic hand grasping different objects. First row: grasping an egg, second row: grasping a calculator, third row: grasping a pair of scissors. First column: hand reference configuration, second column: kinematic manipulability ellipsoids, third column: force manipulability ellipsoids.

efficiency measure. In this paper, the manipulability analysis has been extended to synergy-actuated hands, in which the dimension of the controlled inputs is much lower than the dimension of the contact forces. In this type of manipulation, the compliance has to be taken into account in order to solve the force distribution problem. This paper introduces new manipulability indices which take into account underactuation and compliance. Finally, this more general definition of manipulability is discussed and applied in two examples: the first one is a simple gripper and the second is an anthropomorphic robotic hand.

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Appendix

Quasistatic problem formulation including the derivatives of G, J and S matrices

Let us consider a *variation* with respect to a reference equilibrium condition of the external load $w = w_0 + \delta w$. In the new equilibrium configuration the contact force is $\lambda = \lambda_0 + \delta\lambda$, the joint torques are $\tau = \tau_0 + \delta\tau$, the synergy forces are $\sigma = \sigma_0 + \delta\sigma$. According to Eqs. (5), (7) and (9), and taking into account also the

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G, J and S matrix variations, the following relationships between force and torque variation hold:

$$\delta w + G\delta\lambda + K_g\delta u = 0 \tag{31}$$

$$\delta\tau - J^{\mathrm{T}}\delta\lambda - K_\tau\delta q = 0 \tag{32}$$

$$\delta\sigma - S^{\mathrm{T}}\delta\tau - K_\sigma\delta z = 0 \tag{33}$$

where $K_g = \left.\frac{\partial G\lambda}{\partial u}\right|_0 \in \Re^{n_d\times n_d}$ takes into account the variation of G matrix due to the object displacement, $K_\tau = \left.\frac{\partial J^{\mathrm{T}}\lambda}{\partial q}\right|_0 \in \Re^{n_q\times n_q}$ takes into account the variation of J matrix due to hand joint displacement, $K_\sigma = \left.\frac{\partial S^{\mathrm{T}}\tau}{\partial z}\right|_0 \in \Re^{n_z\times n_z}$ takes into account the variation of S matrix due to the synergy variation, this term has to be considered if the synergy matrix is not constant, i.e. if the relationship between the reference joint position q_r and the input synergies z is not linear. By substituting Eq. (2) into Eq. (32) we can express $\delta\tau$ as

$$\delta\tau = (I - K_\tau C_q)^{-1}J^{\mathrm{T}}\delta\lambda - (I - K_\tau C_q)^{-1}K_\tau S\delta z \tag{34}$$

that can be substituted into Eq. (33), leading to

$$\begin{aligned}&\delta\sigma - S^{\mathrm{T}}(I - K_\tau C_q)^{-1}J^{\mathrm{T}}\delta\lambda \\ &\quad + (S^{\mathrm{T}}(I - K_\tau C_q)^{-1}K_\tau S - K_\sigma)\delta z = 0.\end{aligned} \tag{35}$$

From Eq. (3), taking into account Eqs. (32) and (4), the following expression can be found, relating contact force variation to the synergy variation and object displacement:

$$\delta\lambda = K_{\mathrm{tot}}(J_R S\delta z - G^{\mathrm{T}}\delta u) \tag{36}$$

where

$$K_{\mathrm{tot}} = (C_s + J_R C_q J^{\mathrm{T}}) \tag{37}$$

and $J_R = J(I + C_q K_\tau)^{-1}$. The system defined by Eqs. (31), (35) and (36) can be rewritten as

$$\begin{bmatrix} I & 0 & G & 0 & K_g \\ 0 & I & J_T & K_z & 0 \\ 0 & 0 & I & -K_{\mathrm{tot}}J_R S & K_{\mathrm{tot}}G^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \delta w \\ \delta\sigma \\ \delta\lambda \\ \delta z \\ \delta u \end{bmatrix} = 0 \tag{38}$$

where $K_z = S^{\mathrm{T}}(I - K_\tau C_q)^{-1}K_\tau S$ and $J_T = S^{\mathrm{T}}(I - K_\tau C_q)^{-1}J^{\mathrm{T}}$.

All possible coordinated forces and motion actions of the system belongs to the nullspace of the constraint matrix on the left-hand term of Eq. (38). All the solutions of Eq. (38) can be written as linear combinations of vectors forming a basis of such nullspace. By suitable algebraic manipulation, such a basis can always be written in a block-partitioned form as described in Eq. (17). The considerations on manipulability indices described in Section 4 can then be straightforwardly extended to this more general case.

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