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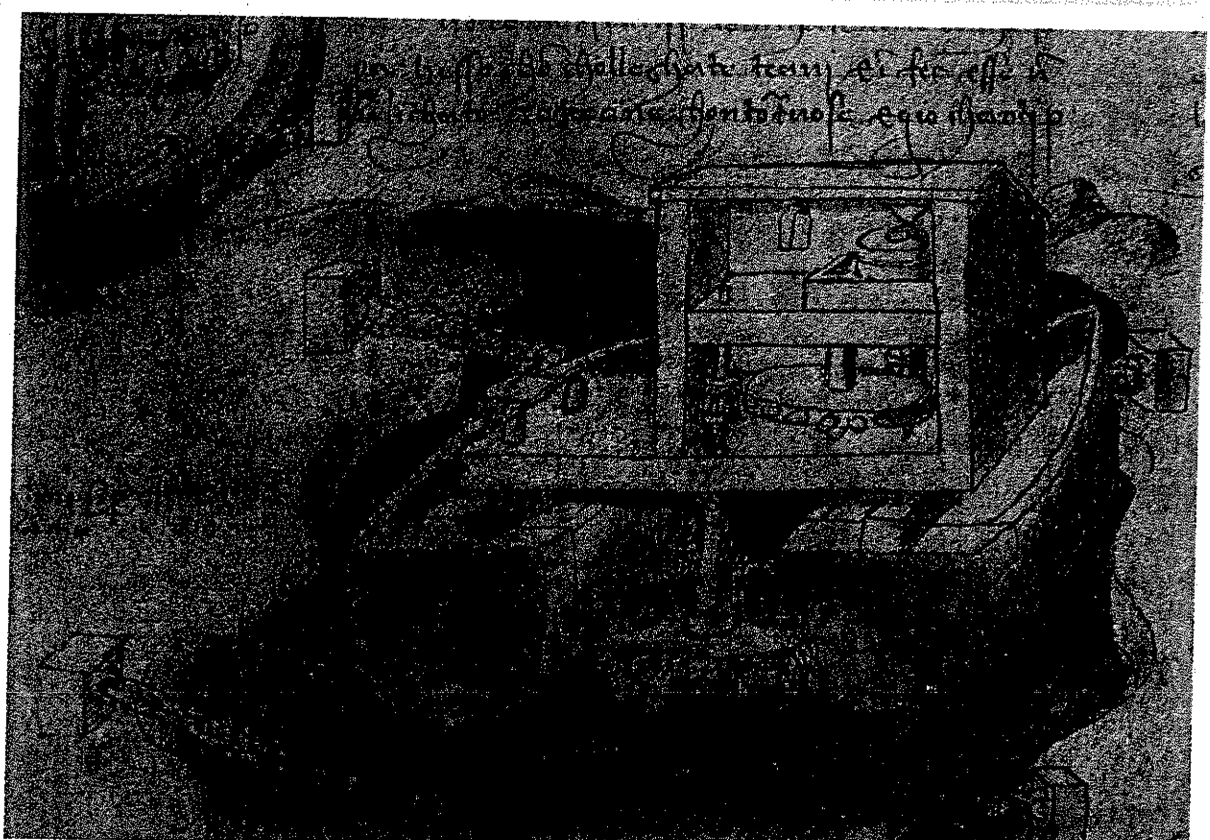

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GEOMETRIC CONTROL TECHNIQUES FOR MECHANICAL SYSTEMS

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ABSTRACT
The aim of the paper consists in showing the effectiveness of geometric control techniques when used to solve several control problems occurring in mechanical applications. The problems of noninteracting control of robotic manipulation systems and disturbance rejection on vehicles with active suspensions are investigated.

SOMMARIO
Le tecniche proprie dell'approccio geometrico al controllo dei sistemi dinamici sono impiegate per la soluzione di alcuni problemi legati al controllo dei sistemi meccanici quali il controllo noninteragente dei sistemi di manipolazione robotica e la reiezione dei disturbi nei veicoli con sospensioni attive.

1 INTRODUCTION

Recently, the geometric approach to the dynamic system and control theory has achieved relevant results which made this approach a powerful tool for the analysis and synthesis of mechanisms with linear and nonlinear dynamics [1, 10, 2].

The class of mechanisms, the paper deals with, is enough general [3] to include both simple robotic end-effectors as the parallel grippers, and more complex devices as serial kinematic chains which may cooperate to manipulate a given object. The class of general manipulation systems is characterized by the following properties:
1. one or more single kinematic chains, consisting of one or more rigid links joined by rotoidal or prismatic joints, are allowed;
2. contacts with the external environment on one or more links of the mechanical structures are allowed;
3. some of the joints may not be actuated.

Figure 1 reports some examples of general manipulation systems.

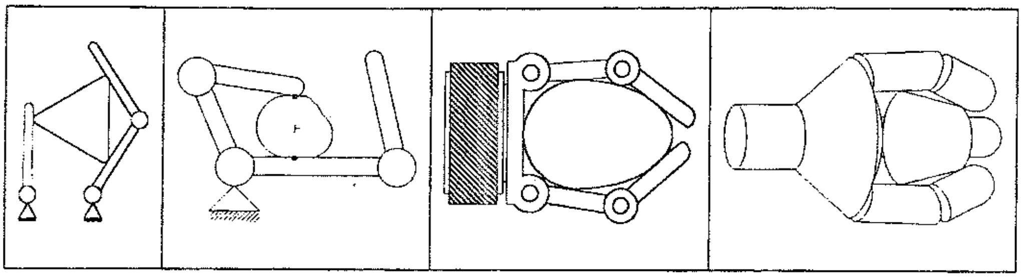

Figure 1: Examples of general manipulation systems.

The task of manipulating parts in robotics can be decomposed in two fundamental sub-tasks. The first one is the control of internal forces or grasping action: the mechanical device must guarantee a stable grasp of the objects in spite of external disturbing forces and/or moments. The second sub-task is the control of the object dynamics: if, for instance, the manipulated objects is a cutting tool, the grasping mechanisms must control the tool trajectory and its cutting force, i.e. the tool dynamics. Notice that, in both sub-tasks, the control of the contact forces between the mechanism and the manipulated object play a fundamental role.

In [5] the authors showed that the control of the object grasp and of its motion are tasks which result to be inherently coupled. The *force/motion decoupling control* of general manipulation systems is studied in the first part of the paper. It should be remarked that the noninteraction requirement is mandatory in robotic assisted surgery [4].

The second control problem, here investigated, is the regulation of the chassis posture in a vehicle equipped with active suspensions. The related control topic corresponds to the problem of localizing unaccessible disturbances [9].

The whole paper is aimed at showing how geometric control tools can be systematically used to obtain the noninteraction and disturbance localization properties in robotic manipulation and vehicle with active suspensions, respectively.

2 MANIPULATION SYSTEMS: NONINTERACTION

The linearized model of the dynamics of a general manipulation system is derived in [3, 4]. Some of those results are here reported for the reader's convenience. Denote by $\mathbf{q} \in \mathbb{R}^q$ the vector of joint positions, $\tau \in \mathbb{R}^q$ the vector of joint actuator torques and $\mathbf{u} \in \mathbb{R}^d$ the vector locally describing the position and the orientation of a frame attached to the external environment (manipulated object).

The force interaction $\mathbf{t}_i$ at the i-th contact is taken into account by using a lumped-parameter $(\mathbf{K}_i, \mathbf{B}_i)$ model of visco-elastic phenomena. According to this model, the vector $\mathbf{t}$, obtained by grouping all contact vectors, is

$$\mathbf{t} = \mathbf{K}({}^h\mathbf{c} - {}^o\mathbf{c}) + \mathbf{B}({}^h\dot{\mathbf{c}} - {}^o\dot{\mathbf{c}}) \tag{1}$$

where ${}^h\mathbf{c}$ and ${}^o\mathbf{c}$ describe the postures of contact frames where the contact spring and damper are anchored. Matrices $\mathbf{K}$ and $\mathbf{B}$ are symmetric and positive definite and their dimension depend on the particular model used to describe the contact interaction, [8].

The kinematic description of the manipulation system is described by the *Jacobian* $\mathbf{J}$ and by the *grasp* matrix $\mathbf{G}$: the linear maps relating the velocities of contact points (on the manipulator and the object) with the joint and object velocities $\dot{\mathbf{q}}$ and $\dot{\mathbf{u}}$ (${}^h\dot{\mathbf{c}} = \mathbf{J}\dot{\mathbf{q}}$, ${}^o\dot{\mathbf{c}} = \mathbf{G}^T\dot{\mathbf{u}}$).

The nonlinear dynamics of the manipulation mechanism and of the object, coupled by the contact force equation (1) are linearly approximated in the neighbourhood of an equilibrium point by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau \delta\tau, \tag{2}$$

where state and input vectors are defined as the departures from the reference equilibrium:

$$\mathbf{x} \; = \; \left[(\mathbf{q}-\mathbf{q}_o)^T \; (\mathbf{u}-\mathbf{u}_o)^T \; \dot{\mathbf{q}}^T \; \dot{\mathbf{u}}^T\right]^T ;$$

and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \; \mathbf{B}_\tau \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}. \tag{3}$$

Assuming that local variations of the Jacobian, grasp matrix and gravity effects are negligible, simple expressions are obtained for blocks $\mathbf{L}_k$ and $\mathbf{L}_b$: $\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k$, and $\mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b$ where $\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o)$ is the inertia matrix of the system,
$\mathbf{P}_k = \left[\mathbf{J} \; -\mathbf{G}^T\right]^T \mathbf{K} \left[\mathbf{J} \; -\mathbf{G}^T\right]$ and $\mathbf{P}_b = \left[\mathbf{J} \; -\mathbf{G}^T\right]^T \mathbf{B} \left[\mathbf{J} \; -\mathbf{G}^T\right]$.

2.1 FORCE/MOTION NONINTERACTION

In robotic manipulation the controlled outputs are typically the *internal* or *grasping* forces which belong to the null space of the grasp matrix $\mathbf{G}$ and the rigid-body object kinematics deeply discussed in [3]. A special subspace of internal forces and of the rigid-body object motions are now characterized through output matrices of the linearized dynamics of manipulation mechanisms: the *reachable internal* contact forces $\mathbf{t}_i$, defined as the projection of the force vector $\mathbf{t}$ onto the null space of $\mathbf{G}$,

$$\mathbf{t}_i = \mathbf{E}_{ti}\mathbf{x} = \left\{(\mathbf{Q}^T\mathbf{Q})^{-1}\mathbf{Q}^T\left[\mathbf{Q}\;\mathbf{0}\;\mathbf{Q}\;\mathbf{0}\right]\right\}\mathbf{x}, \quad \mathbf{Q} = (\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J} \tag{4}$$

and the *rigid-body* object motions $\mathbf{u}_c$ defined as the projection of the object displacement $\mathbf{u}$ onto the column space of a matrix Γ_{uc} (satisfying $\mathbf{J}\Gamma_{qc} = \mathbf{G}^T\Gamma_{uc}$):

$$\mathbf{u}_c = \mathbf{E}_{uc}\mathbf{x}, \quad \mathbf{E}_{uc} = (\Gamma_{uc}^T\Gamma)^{-1}\Gamma_{uc}^T\left[\mathbf{0}\;\mathbf{I}\;\mathbf{0}\;\mathbf{0}\right]. \tag{5}$$

The following decoupling theorem has been proved in a geometric framework [5]. It states that a noninteracting controller exists such that internal forces $\mathbf{t}_i$ can be controlled without affecting the object motions $\mathbf{u}_c$ and *vice versa*.

Theorem 1 (Noninteraction) *Consider the linearized manipulation system of Section 2. If* $ker(\mathbf{G}^T) = \{\mathbf{0}\}$*, there exists a stabilizing state-feedback control law,* $\tau = \mathbf{F}\mathbf{x} + \tau^*$ *and an input partition* $\tau^* = \mathbf{U}_{ti}\mathbf{u}_{ti} + \mathbf{U}_{uc}\mathbf{u}_{uc}$ *which decouples reachable internal forces* $\mathbf{t}_i$ *and rigid-body object motions* $\mathbf{u}_c$.

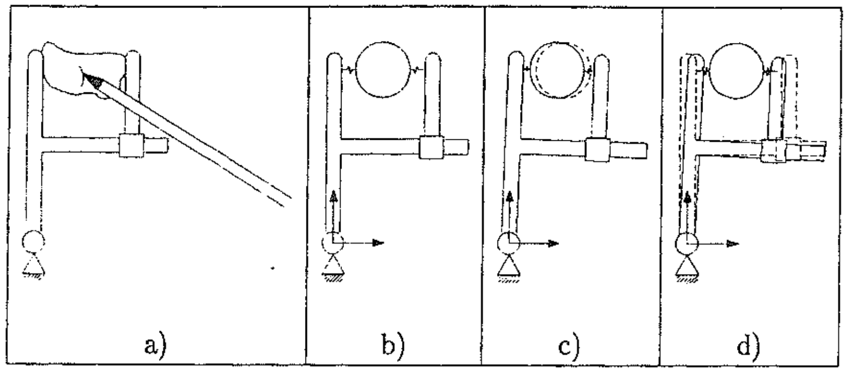

Figure 2: (a) Robotic assisted surgery; (b) Lumped parameter model; (c) Typical internal force control; (d) *Nonintercting* control.

Theorem 1 states that a state–feedback control law and a joint torques partition exists such that, for zero initial conditions, each input only affects the relative output. The geometric concept from which the previous result develops is the *$\mathcal{S}$–constrained controllability*. It consists of those state space vectors reachable through trajectories entirely lying in the constraining subspace $\mathcal{S}$.

The importance of the noninteracting control law control is highlighted by the following example. Consider the manipulation system of fig.2–a and model the complaint contacts with visco–elastic lumped parameters as depicted in fig.2–b. A control action aimed to increase the grasping force, but which does not takes into account the non-interaction, squeezes the manipulated object but gives rise to undesired and dangerous transient motion of the object (fig 2–c). On the contrary the noninterating control suggested in this paper completely decouples the control of internal forces from the object dynamics as depicted in (fig 2–d).

3 ACTIVE SUSPENSIONS: DISTURBANCE LOCALIZATION

The second part of the paper presents an application of the geometric control theory to the problem of localizing disturbances of a vehicle with active suspensions for the regulation of the chassis posture. It will be shown that the regulated variables, i.e. the roll and pitch angles and the chassis heights, can be decoupled from the external disturbances by means of a state feedback control law.

The mathematical model of vertical dynamics of road vehicles is derived for the mechanical structure reported in figure 3. The vehicle sprung mass is linked with the axes by means of four passive suspensions and actuators. An independent control action u_j ($j = 1, \ldots, 4$) is exerted at each corner of the vehicle.

The section focuses on the regulation of the chassis posture in spite of disturbances d_j transmitted through the suspensions (*ride heights regulation*, [9]). Thus the controlled output vector

$$\mathbf{y} = (\theta_r,\ \theta_p,\ z)^T. \tag{6}$$

consists of the roll, pitch angles and of the vehicle height, cf. fig. 3.

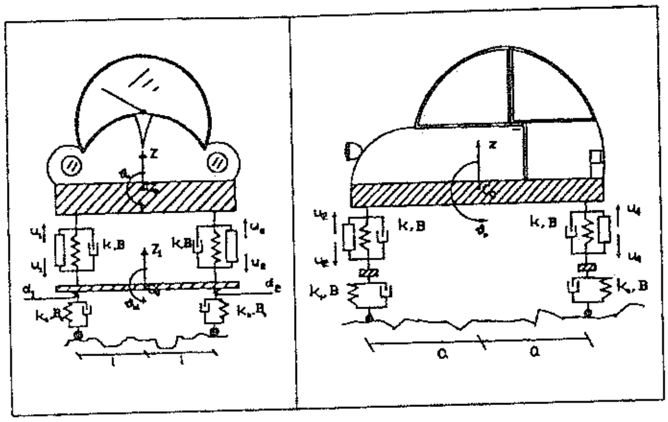

Figure 3: Mechanical model of a vehicle with active suspensions, front and side view. $\theta_r, \theta_p, \theta_{ai}, z$ and z_i are roll, pitch and i-th axis angles, chassis height and i-th axis height, respectively.

The vertical dynamics is linearized around an equilibrium configuration and can be described in the state space as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d}; \\ \mathbf{y} = \mathbf{C}\mathbf{x}, \end{cases} \tag{7}$$

where the 4-dimensional input and disturbance vector are $\mathbf{u} = (u_1\ u_2\ u_3\ u_4)$ and $\mathbf{d} = (d_1\ d_2\ d_3\ d_4)$, and the 14-dimensional state vector is $\mathbf{x} = (\mathbf{x}_r^T \mathbf{x}_v^T)^T$ being (cf. fig. 3),

$$\mathbf{x}_r = (\theta_r\ \theta_{a1}\ \theta_{a2}\ \dot{\theta}_r\ \dot{\theta}_{a1}\ \dot{\theta}_{a2})^T; \quad \mathbf{x}_v = (\theta_p\ z\ z_1\ z_2\ \dot{\theta}_p\ \dot{z}\ \dot{z}_1\ \dot{z}_2)^T.$$

For the sake of brevity matrices of dynamics model have not been reported. A deep analysis of these matrices can be found in [7].

3.1 DISTURBANCE LOCALIZATION

On the base of previous formulation, the *ride heights regulation* can be rigorously stated as a problem of unaccessible disturbance localization. The problem consists in finding a state feedback $\mathbf{u} = \mathbf{F}\mathbf{x}$ for the dynamic system (7), such that, starting at zero state, the regulated output $\mathbf{y}(\mathbf{t})$ is identically zero for all the admissible disturbances $\mathbf{d}(\mathbf{t})$.

The problem is attacked by using the classical tools of the geometric control theory. It is well known that the unaccessible disturbance localization problem has a solution if and only if there exists a matrix $\mathbf{F}$ such that $\min \mathcal{I}(\mathbf{A}+\mathbf{BF}, \mathbf{D})$, the minimal $(\mathbf{A}+\mathbf{BF})$-invariant subspace containing the column space of the disturbance matrix $\mathbf{D}$, is included in the nullspace of the regulated output matrix, $ker(\mathbf{C})$. Since this condition depends on the choice of $\mathbf{F}$, it lacks convenience and an equivalent structural condition [1] is preferred to prove the following theorem.

Theorem 2 (Disturbance localization.) *For the dynamic system (7) of a vehicle with active suspensions, there always exists a stabilizing state feedback gain* $\mathbf{F}$ *which localizes disturbances* $\mathbf{d}$ *in the nullspace of the regulated output* $\mathbf{y} = (\theta_r, \theta_p, z)$.

The theorem shows that the unaccessible disturbance localization for the regulated output $\mathbf{y}$ of the dynamic system, is an intrinsic structural property of vehicles with active suspensions. In other words there always exists a static state feedback control law able to decouple the dynamic behaviour of the chassis height from external disturbances $\mathbf{d}$.

4 CONCLUSIONS

Classical results of the geometric approach to the analysis and control of linear dynamic systems have been applied to complex mechanical structures such as the general manipulation systems and road vehicles equipped with active suspensions.

Due to the generality of the approach the force/motion nointeracting control in manipulation systems and the disturbance localization in vehicles can be ranked as structural properties of these mechanisms.

Future perspectives of the work concern the extension of these results to the full nonlinear model of mechanisms dynamics.

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