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Specifying Consistent Control Goals for Kinematically Defective Manipulation Systems

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Abstract

In this paper, we focus on the problem of controlling a manipulator so as to track a desired object trajectory, while guaranteeing that contact forces comply with contact constraints (friction bounds, etc.). When dealing with kinematically defective systems, it is not possible in general to assign arbitrary trajectories of object motions and contact forces. To understand what restrictions position and force reference trajectories should exhibit in order to be feasible by a given system, is the central issue of this work.

1 Introduction

The peculiarity of kinematically defective manipulation systems consists of the presence of parts, interacting with the environment, that have fewer degrees-of-freedom than those necessary to achieve arbitrary configurations in their operational space. Such systems occur for instance when dealing with simple industry-oriented grippers, as illustrated in fig. 1; or when the whole surface of the manipulator limbs is exploited to constrain the manipulated object, such as e.g. in tentacle-like arms or in "whole-arm" manipulation ([10], [2], [7], [11]), see fig. 2. In general, kinematic defectivity arises very often when an attempt is made at minimizing the mechanical hardware of the manipulator system. (such as e.g. in [5], [8]).

Our focus here is on the problem of tracking a desired trajectory with the manipulated object, while guaranteeing that contact forces are controlled so as to comply with contact constraints at every instant. For kinematically defective systems, this problem is not solvable in general for arbitrarily assigned trajectories. In the most simple example provided in fig. 3, not all trajectories of the object can be controlled in the plane, nor can arbitrary contact forces be applied on the object. Understanding what characteristics required trajectories should have in order to be feasible by a given system is therefore crucial to the design of any planning and control algorithm for these systems.

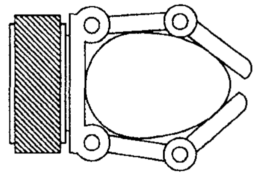

Figure 1: A two-fingered 2 d.o.f. gripper with curling fingers grasping an object.

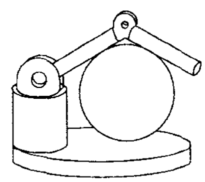

Figure 2: Robust hold of an object by means of an enveloping, "whole-arm" grasp.

The main result of this paper, stated in theorem 1, provides a geometric description and an algorithm for evaluating a set of locally feasible trajectories of motions and forces.

The local nature of our results is due to the linearization approach of the dynamics that is used in this work. The use of linearized model dynamics in the analysis of general manipulation systems is believed to be a significant advancement with respect to the literature, which is almost solely based on quasi-static models, and in fact provides richer results and better insight. Furthermore, linearized analysis is considered as a fundamental preparatory step towards full nonlinear analysis, which at the moment appears to be too complex to achieve in full generality.

Such an approach has been motivated both by the higher readability of the achieved results (w.r.t. the nonlinear ones) and by the fact that the linearization

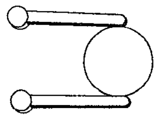

Figure 3: Not all the object trajectories can be controlled.

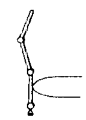	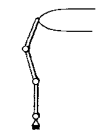	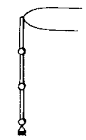
$\ker(\mathbf{J}^T) \neq \mathbf{0}$	$\ker(\mathbf{J}) \neq \mathbf{0}$ ($\ker(\mathbf{J}^T) = \mathbf{0}$)	$\ker(\mathbf{J}^T) \neq \mathbf{0}$

Table 1: 3–joint planar manipulation: defective, redundant (non–defective) and singular configurations.

approach is with no doubt more complete of the quasi–static one, typically used in this field.

2 Dynamic model

The starting point of our analysis is the linearized model of the dynamics of general manipulation systems derived in [9] (see also [3]). In this section we summarize some of those results for the reader's convenience. We denote by $\mathbf{q} \in \mathbb{R}^q$ the vector of manipulator joint positions, $\tau \in \mathbb{R}^q$ the vector of joint actuator torques, $\mathbf{u} \in \mathbb{R}^d$ the vector locally describing the position and the orientation of a frame attached to the object and finally $\mathbf{w} \in \mathbb{R}^d$ the vector of forces and torques resultant from external forces acting directly on the object. Let further introduce the vector $\mathbf{t}$ (of dimension t) in which all contact force vectors exchanged at the contacts between the links and the object are juxtaposed. We assume that contact forces arise from a lumped–parameter model of visco–elastic phenomena at the contacts, summarized by a stiffness matrix $\mathbf{K}$ and damping matrix $\mathbf{B}$. The Jacobian matrix $\mathbf{J}$ and the grasp matrix $\mathbf{G}$ of the manipulation system are defined as usual as the linear maps relating the velocities of the contact points on the links and on the object, with the joint and object velocities, respectively. Consider a reference equilibrium configuration ($\mathbf{q} = \mathbf{q}_o, \mathbf{u} = \mathbf{u}_o, \dot{\mathbf{q}} = \dot{\mathbf{u}} = \mathbf{0}, \tau = \tau_o, \omega = \omega_o$ and $\mathbf{t} = \mathbf{t}_o$), such that $\tau_o = \mathbf{J}^T\mathbf{t}_o$ and $\mathbf{w}_o = -\mathbf{G}\mathbf{t}_o$. The linear approximation of the manipulation system in the neighborhood of such equilibrium is written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau \tau' + \mathbf{B}_w \mathbf{w}', \tag{1}$$

where state and input vectors are defined as the departures from the reference equilibrium configuration: $\mathbf{x} = \left[(\mathbf{q}-\mathbf{q}_o)^T \; (\mathbf{u}-\mathbf{u}_o)^T \; \dot{\mathbf{q}}^T \; \dot{\mathbf{u}}^T\right]^T, \tau' = \tau - \mathbf{J}^T\mathbf{t}_o$ and $\mathbf{w}' = \mathbf{w} + \mathbf{G}\mathbf{t}_o$, and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \; \mathbf{B}_\tau = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \; \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix}.$$

To simplify notation we will henceforth omit the apexes in τ' and $\mathbf{w}'$.

Neglecting gravity, assuming a locally isotropic model of viscoelastic phenomena, and assuming that local variations of the jacobian and grasp matrices are small, simple expressions are obtained for $\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k$ and $\mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b$, where $\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o)$, $\mathbf{P}_k = \mathbf{S}^T\mathbf{K}\mathbf{S}$, $\mathbf{P}_b = \mathbf{S}^T\mathbf{B}\mathbf{S}$, and $\mathbf{S} = [\mathbf{J} \; -\mathbf{G}^T]$.

To our purposes, three possible combinations of states are of interest as outputs, namely object positions, joint positions, and forces. The corresponding output matrices are, respectively,

$$\begin{aligned} \mathbf{C}_u &= [\; \mathbf{0} \;\; \mathbf{I} \;\; \mathbf{0} \;\; \mathbf{0} \;]; \\ \mathbf{C}_q &= [\; \mathbf{I} \;\; \mathbf{0} \;\; \mathbf{0} \;\; \mathbf{0} \;]; \\ \mathbf{C}_t &= [\; \mathbf{K}\mathbf{J} \;\; -\mathbf{K}\mathbf{G}^T \;\; \mathbf{B}\mathbf{J} \;\; -\mathbf{B}\mathbf{G}^T \;]. \end{aligned}$$

A few definitions are useful for general manipulation systems (see [3]),

Definition 1 *A manipulation system is said "defective" if* $\ker(\mathbf{J}^T) \neq \mathbf{0}$*; "indeterminate" if* $\ker(\mathbf{G}^T) \neq \mathbf{0}$*; "redundant" if* $\ker(\mathbf{J}) \neq \mathbf{0}$*, "graspable" if* $\ker(\mathbf{G}) \neq \mathbf{0}$ *and "hyperstatic" if* $\ker(\mathbf{J}^T) \cap \ker(\mathbf{G}) \neq \mathbf{0}$

Observe that the manipulation system is kinematically defective if at least one of the links touching the object possesses less degrees–of–freedom than those necessary to move its contact point in arbitrary directions, or, in terms of forces, if there exists at least one direction of the contact vector $\mathbf{t}$ that does not affect the manipulator dynamics. Whenever the number t of components of contact forces is larger than the number q of joints, the system is defective. Note also that if the manipulator is in a singular configuration, $\ker(\mathbf{J}^T) \neq \mathbf{0}$ as well. Table 1 pictorially illustrates such definitions.

3 Stability and stabilizability

We consider some aspects related to the analysis of the stability of the linearized model of a manipulation and grasping system. The characteristic polynomial of the linearized system is: $\det(s\mathbf{I} - \mathbf{A}) = \det(s^2\mathbf{M} + s\mathbf{P}_b + \mathbf{P}_k)$. Recalling that $\mathbf{M}$ is positive definite (p.d.) and noting that $\mathbf{P}_k$, $\mathbf{P}_b$ are positive semidefinite (p.s.d.) matrices, the eigenvalues of the linearized system lie in the union of the open left–half–plane and the origin.

Due to the existence of a nullspace of the dynamic matrix $\mathbf{A}$, no conclusion can be drawn about the local stability of the full, nonlinear dynamics about the equilibrium configuration However, the following *restricted stabilizability* lemma is of practical relevance:

Lemma 1 *The dynamics of a non–indeterminate manipulation system, i.e.* $\mathbf{G}$ *is full row rank (f.r.r.), is made locally asymptotically stable by a constant linear state feedback of joint displacements and rates only, with feedback matrix* $\mathbf{R}' = \begin{bmatrix} \mathbf{R}_q & 0 & \mathbf{R}_{\dot{q}} & 0 \end{bmatrix}$ *provided that* $\mathbf{R}_q$ *and* $\mathbf{R}_{\dot{q}}$ *are p.d. matrices.*

proof: Recall that $\det(s\mathbf{I} - (\mathbf{A} - \mathbf{B}_\tau \mathbf{R})) = \det(s^2\mathbf{M} + s\mathbf{P}'_b + \mathbf{P}'_k)$, where $\mathbf{P}'_k = \mathbf{P}_k + \text{diag}(\mathbf{R}_q, \mathbf{0})$ and $\mathbf{P}'_b = \mathbf{P}_b + \text{diag}(\mathbf{R}_{\dot{q}}, \mathbf{0})$. Being $\mathbf{P}'_k$, $\mathbf{P}'_b$ p.d., putting $\mathbf{K} = \mathbf{K}^{T/2}\mathbf{K}^{1/2}$, we have that $\mathbf{x}^T\mathbf{P}'_k\mathbf{x} = (\mathbf{K}^{1/2}\mathbf{J}\mathbf{x}_1 - \mathbf{K}^{1/2}\mathbf{G}^T\mathbf{x}_2)^T(\mathbf{K}^{1/2}\mathbf{J}\mathbf{x}_1 - \mathbf{K}^{1/2}\mathbf{G}^T\mathbf{x}_2) + \mathbf{x}_1^T\mathbf{R}_q\mathbf{x}_1 > 0$, and analogously for $\mathbf{P}'_b$.□

4 Functional controllability

As already pointed out, we are interested in the problem of following a desired trajectory with the manipulated object, while guaranteeing that contact forces are controlled so as to comply with contact constraints at every instant. In system theory this problem is known as "functional controllability". Although functional controllability is generally approached by state–space methods, for linear systems it is most simply studied in terms of input–output representations. A well–known necessary and sufficient condition for the output functional controllability of linear system is reported in the following proposition

Proposition 1 *Let* $\mathbf{Z}(s)$ *be the* $(d \times q)$ *transfer function matrix of a given linear system. A necessary and sufficient condition for the functional controllability of* d *arbitrary smooth* (C^∞) *outputs by* q *smooth inputs is that the transfer function matrix* $\mathbf{Z}(s)$ *is f.r.r. over the field of complex numbers.*

Explicitly note that the output functional controllability requires that at least as many inputs are available as there are outputs of concern.

Consider the linearized model in sec. 2 with feedback $\mathbf{R}_q$ from joint positions and $\mathbf{R}_{\dot{q}}$ from joint velocities. $\hat{\mathbf{A}}$ and τ will henceforth indicate the dynamic matrix with feedback and the reference input, respectively. Let $\mathbf{u}$ be the system output, in the Laplace domain the input–output representation is $\mathbf{u}(s) = \mathbf{Z}_{u,\tau}(s)\tau(s) + \mathbf{Z}_{u,w}\mathbf{w}(s)$, with $\mathbf{Z}_{u,\tau} = -\mathcal{D}^{-1}\mathcal{B}^T\mathcal{X}$ and $\mathbf{Z}_{u,w} = (\mathcal{D} - \mathcal{B}^T\mathcal{A}^{-1}\mathcal{B})^{-1}$ where

$$\mathcal{A} = s^2\mathbf{M}_h + s(\mathbf{J}^T\mathbf{B}\mathbf{J} + \mathbf{R}_{\dot{q}}) + \mathbf{J}^T\mathbf{K}\mathbf{J} + \mathbf{R}_q;$$
$$\mathcal{B} = -s\mathbf{J}^T\mathbf{B}\mathbf{G}^T - \mathbf{J}^T\mathbf{K}\mathbf{G}^T;$$
$$\mathcal{D} = s^2\mathbf{M}_o + s\mathbf{G}\mathbf{B}\mathbf{G}^T + \mathbf{G}\mathbf{K}\mathbf{G}^T;$$
$$\mathcal{X} = (\mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{B}^T)^{-1}.$$

Obviously at least d joints are necessary to track arbitrary object trajectories in a d–dimensional space.

Analogous considerations apply for contact forces, $\mathbf{t}(s) = \mathbf{Z}_{t,\tau}(s)\tau(s) + \mathbf{Z}_{t,w}\mathbf{w}(s)$, with $\mathbf{Z}_{t,\tau} = \mathbf{C}_t\ (s\mathbf{I} - \hat{\mathbf{A}})^{-1}\ \mathbf{B}_\tau = (\mathbf{K} + s\mathbf{B})(\mathbf{J}\mathcal{X} - \mathbf{G}^T\mathcal{Z})$ and $\mathbf{Z}_{t,w} = \mathbf{C}_t\ (s\mathbf{I} - \hat{\mathbf{A}})^{-1}\ \mathbf{B}_w = (\mathbf{K} + s\mathbf{B})(\mathbf{J}\mathcal{Y} - \mathbf{G}^T\mathcal{W})$, where $\mathcal{Z} = -\mathcal{D}^{-1}\mathcal{B}^T\mathcal{X}, \mathcal{W} = (\mathcal{D} - \mathcal{B}^T\mathcal{A}^{-1}\mathcal{B})^{-1}$ and $\mathcal{Y} = -\mathcal{A}^{-1}\mathcal{B}\mathcal{W}$. Being $\mathbb{R}^t$ the space of contact forces, in absence of disturbances $\mathbf{w}$, at least t joints are necessary to track arbitrary contact forces. This is not possible in defective systems.

In this paper we will focus on the *definition of a new set of outputs that is functionally controllable* and *relevant to the task of manipulation.* In order to do this, the concept of "asymptotic reproducibility" [4] is instrumental. Asymptotic reproducibility investigates output tracking for a particular class of trajectories, those constant in time. The following definition formalizes the notion of asymptotic reproducibility.

Definition 2 *Let* $\mathbf{y}(s)/\tau(s) = \mathbf{Z}(s)$ *be the transfer matrix of an asymptotically stable system, the subspace of asymptotic reproducibility is defined as the column space of* $\mathbf{Z}(0)$. *The system output is asymptotically reproducible if the gain matrix* $\mathbf{Z}(0)$ *is f.r.r..*

Remark 1. The asymptotic reproducibility of the outputs of an asymptotically stable system is a sufficient condition for the functional reproducibility of the same outputs.

In the sequel, we assume that the manipulation system has no indeterminate modes ($\mathbf{G}$ is f.r.r) and that joint position and rates have been fed back such that all modes of the system are asymptotically stable.

4.1 Contact forces

Under the above assumptions, the steady–state gain matrix for contact forces from joint inputs can be evaluated, after some algebraic manipulation, as

$$\mathbf{Z}_{t,\tau}(0) = -\mathbf{C}_t\hat{\mathbf{A}}^{-1}\mathbf{B}_\tau = -(\mathbf{I} - \mathbf{G}^+_{\bar{K}}\mathbf{G})\bar{\mathbf{K}}\mathbf{J}, \quad (2)$$

where $\mathbf{G}^+_{\bar{K}}$ is the $\bar{\mathbf{K}}$–weighted pseudoinverse of $\mathbf{G}$, and $\bar{\mathbf{K}}^{-1} = \mathbf{K}^{-1} + \mathbf{J}\mathbf{R}_q^{-1}\mathbf{J}^T$ is the equivalent stiffness matrix including the effect of proportional control on joint positions (cf. [6]). The subspace $\mathcal{F}_{hr} = \text{range}(\mathbf{Z}_{t,\tau}(0))$ is defined as the subspace of "asymptotically internal forces" and consists of all the contact forces that are reachable at steady–state. Observe that $\mathcal{F}_{hr} \subseteq \ker(\mathbf{G})$: these forces are self–balanced and their resultant action on the object dynamics is null. In robotic grasp literature, forces $\mathbf{t} \in \ker(\mathbf{G})$ are customarily defined "internal", and play a fundamental role in grasp contact stability. The importance of the controllability of internal forces in grasping was put into evidence in a previous work by Bicchi

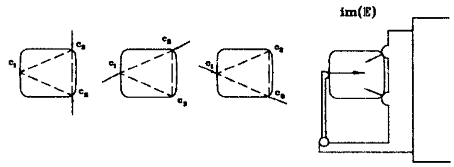

Figure 4: Example of asymptotic reproducible contact forces ($\mathcal{F}_{hr} \subseteq \mathsf{ker}(\mathbf{G})$).

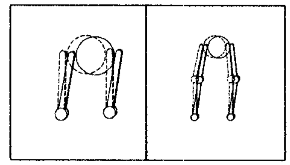

Table 2: Representative motions for the subspace $\mathsf{range}\left(\left[\mathbf{\Gamma}_{qc}^T \; \mathbf{\Gamma}_{uc}^T\right]^T\right)$.

[1] where the principle of virtual work was used in a quasi–static approach to describe the subspace of "active" internal forces. Simple calculations show that such subspace coincides with $\mathcal{F}_{hr}$. The example in figure 4.1 illustrates asymptotically internal contact forces. While the subspace of internal forces ($\mathsf{ker}(\mathbf{G})$) is 3–dimensional, as depicted on the left side of the picture, only a one–dimensional subspace is asymptotically reproducible.

It should be pointed out that, in general, asymptotically reproducible internal forces are internal only at steady–state, and it might not be possible to apply them without a transient phase affecting the equilibrium of the object. Consider for instance the example in fig. 4: when a step of torque is applied at the joint to "squeeze" the object, it causes the motion of the object, which recovers a (displaced) equilibrium only after the transient is finished. In other cases, due to symmetries in the mechanism, it might be possible to apply internal forces that remain such during the transients as well. Such "dynamically internal" forces have been investigated in detail by Prattichizzo [9].

4.2 Object motions

The subspace $\mathcal{U}_r = \mathsf{range}(\mathbf{Z}_{u,\tau}(0))$, where

$$\mathbf{Z}_{u,\tau}(0) = -\mathbf{C_u}\hat{\mathbf{A}}^{-1}\mathbf{B}_\tau = (\mathbf{G}\bar{\mathbf{K}}\mathbf{G}^T)^{-1}\mathbf{G}\bar{\mathbf{K}}\mathbf{J}, \quad (3)$$

is comprised of all asymptotically reproducible displacements of the object from joint torques. In the sequel, it will be shown that every displacement of the object complying with a rigid–body model of the system is asymptotically reproducible.

Rigid–body kinematics are of particular interest in the control of robotic manipulation systems, because the extent to which displacements from the reference equilibrium comply with the linearized model is much limited for motions that involve visco–elastic deformations of bodies. Rigid–body kinematics have been studied in a quasi–static setting [2] and in terms of unobservable subspaces in [3]. In both cases rigid kinematics were described by a matrix $\mathbf{\Gamma}$ whose columns form a basis for $\mathsf{ker}\left[\mathbf{J} \; -\mathbf{G}^T\right]$. In our present assumption that the system is not indeterminate, it is

$$\mathbf{\Gamma} = \mathsf{ker}\left[\begin{array}{cc} \mathbf{J} & -\mathbf{G}^T \end{array}\right] = \left[\begin{array}{cc} \mathbf{\Gamma}_r & \mathbf{\Gamma}_{qc} \\ \mathbf{0} & \mathbf{\Gamma}_{uc} \end{array}\right], \quad (4)$$

where $\mathbf{\Gamma}_r$ is a basis matrix of the subspace of redundant motions $\mathsf{ker}(\mathbf{J})$, and $\mathbf{\Gamma}_{qc}$ and $\mathbf{\Gamma}_{uc}$ are conformal partitions of a complementary basis matrix. The image spaces of $\mathbf{\Gamma}_{qc}$ and $\mathbf{\Gamma}_{uc}$ consist of coordinated rigid–body motions of the mechanism, for the links and the object parts, respectively. Table 2 illustrates such subspaces for two simple devices.

It can be shown that rigid–body coordinate motions of the object are asymptotically reproducible from joint torques,

$$\mathsf{range}(\mathbf{\Gamma}_{uc}) \in \mathsf{range}((\mathbf{G}\bar{\mathbf{K}}\mathbf{G}^T)^{-1}\mathbf{G}\bar{\mathbf{K}}\mathbf{J}) = \mathcal{U}_r. \quad (5)$$

Notice that rigid–body motions are not the only asymptotically reproducible object motions; $\mathcal{U}_r$ also contains motions due to deformations of elastic elements in the model, as for instance, horizontal motions of the object in the device of figure 4.

4.3 Functional controllability of contact forces and object motions

In general, not all the object motions are functionally controllable by joint torques. Object trajectories $\mathbf{u}_{des}$ can be executed if they remain within the subspace $\mathcal{U}_r$ and analogously, arbitrary contact force trajectories $\mathbf{t}_{des}$ can be executed if they evolve within the subspace $\mathcal{F}_{hr}$. In manipulation, however, due to the presence of friction constraints, task specifications can not be given disjointly in terms of either object positions or contact forces.

Clearly, conditions $\mathbf{u}_{des} \in \mathcal{U}_r$ and $\mathbf{t}_{des} \in \mathcal{F}_{hr}$ are only necessary, but no longer sufficient, for *joint* functional controllability of object motions and contact forces. Moreover, specifications of jointly functionally controllable object motions and contact forces may not exhaust the control capabilities of the system.

Our goal is therefore to define a set of outputs for a general manipulation systems that is guaranteed to be feasible, that fully exploits the control inputs and that is convenient for the specification of the tasks. The first requirement implies that the new outputs are functionally controllable; the second that the input–output system is square and the third that the new outputs incorporate the typical priorities of a manipulation task with its priorities:

a) object trajectories that can be accommodated for by the mechanism;

b) contact forces that can be steered so as to avoid violation of contact constraints;

c) reconfiguration of limbs in presence of redundancy.

The following theorem proposes a functionally controllable and task–oriented set of outputs for general manipulation systems

Theorem 1 *In the hypothesis that* $\ker(\mathbf{G}^T) = \mathbf{0}$, *consider the linearized dynamics described by the triple* $(\mathbf{A}, \mathbf{B}_\tau, \mathbf{C})$, *where* $\mathbf{A}$ *and* $\mathbf{B}_\tau$ *are as in sec. 2, and the output matrix* $\mathbf{C}$ *is defined as*

$$\mathbf{C} = \begin{bmatrix} \mathbf{\Gamma}_{uc}^{+}\mathbf{C_u} \\ \mathbf{E}^{+}\mathbf{C_t} \\ \mathbf{\Gamma}_r^{+}\mathbf{C_q} \end{bmatrix}, \qquad (6)$$

where $\mathbf{\Gamma}_r$ *and* $\mathbf{\Gamma}_{uc}$ *have been defined in (4), and* $\mathbf{E}$ *is a basis matrix for* $\mathcal{F}_{hr}$. *Then, for any constant linear state feedback* $\mathbf{R}$ *such that* $\mathbf{A} - \mathbf{B}_\tau\mathbf{R}$ *is asymptotically stable, the system* $(\mathbf{A} - \mathbf{B}_\tau\mathbf{R},\ \mathbf{B}_\tau,\ \mathbf{C})$ *is square and functionally controllable.*

proof: Note that the existence of such feedback matrix $\mathbf{R}$ is guaranteed by lemma 1.

a) The system is square if the number of columns of $\mathbf{C}^T$, denoted by $\#(\mathbf{C})$, is equal to the input space dimension, i.e. if $\#(\mathbf{\Gamma}_{uc}) + \#(\mathbf{E}) + \#(\mathbf{\Gamma}_r) = q$. Since $\mathbf{\Gamma}_r$, $\mathbf{\Gamma}_{uc}$, and $\mathbf{E}$ are f.c.r. by definition, from (4) we have $\#(\Gamma_{uc}) + \#(\Gamma_r) = \dim(\ker([\mathbf{J}\ \ -\mathbf{G}^T])) - \dim(\ker(\mathbf{G}^T))$. Observing that $\ker((\mathbf{I} - \mathbf{G}_K^R\mathbf{G})) = \mathrm{range}(\mathbf{K}\mathbf{G}^T)$, from (2) we obtain

$$\#(\mathbf{E}) = \#(\mathbf{J}) - \dim(\ker(\mathbf{J})) - \dim(\mathrm{range}(\mathbf{J}) \cap \mathrm{range}(\mathbf{G}^T)) =$$
$$= q - \dim(\ker(\mathbf{J})) - \Big[\dim(\ker([\mathbf{J}\ \ -\mathbf{G}^T]))$$
$$- \dim(\ker(\mathbf{J})) - \dim(\ker(\mathbf{G}^T))\Big] = q - \#(\Gamma_r) - \#(\Gamma_{uc});$$

b) To prove output functional controllability of the system $(\mathbf{A} - \mathbf{B}_\tau\mathbf{R},\ \mathbf{B}_\tau,\ \mathbf{C})$, it will be shown that $\mathbf{Z}_C(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A} + \mathbf{B}_\tau\mathbf{R})^{-1}\mathbf{B}_\tau$ has rank q over the complex field. The steady–state gain matrix $\mathbf{Z}_C(0)$ results

$$\mathbf{Z}_C(0) = \begin{bmatrix} \mathbf{\Gamma}_{uc}^{+}(\mathbf{GKG}^T)^{-1}\mathbf{GKJ} \\ \mathbf{E}^{+}(\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ} \\ \mathbf{\Gamma}_r^{+} \end{bmatrix} \Xi,$$

where $\Xi = -\left(\mathbf{J}^T\mathbf{KJ} + \mathbf{R}_q - (\mathbf{J}^T\mathbf{KG}^T - \mathbf{R}_u)(\mathbf{GKG}^T)^{-1}\mathbf{GKJ}\right)^{-1}$.

From remark 1, the f.r.r. of $\mathbf{Z}_C(0)$ is a sufficient condition for functional controllability, that can be shown by proving that $\ker(\mathbf{Z}_C(0)^T) = \mathbf{0}$.

Transposing $\mathbf{Z}_C(\mathbf{0})$, we get that

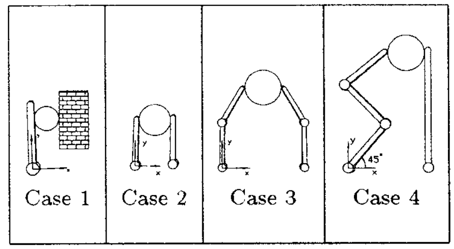

Table 3: Four simple planar manipulators.

$$\ker(\mathbf{Z}_C(0)^T) = \ker \begin{bmatrix} \mathbf{\Gamma}_{uc}^{+}(\mathbf{GKG}^T)^{-1}\mathbf{GKJ} \\ \mathbf{E}^{+}(\mathbf{I} - \mathbf{G}_K^{+}\mathbf{G})\mathbf{KJ} \\ \mathbf{\Gamma}_r^{+} \end{bmatrix}^T.$$

Observe that each row block of the matrix on the right–hand side of equation above is f.c.r., in fact

i: $\mathrm{range}(\mathbf{\Gamma}_{uc}) \subseteq \mathrm{range}((\mathbf{GKG}^T)^{-1}\mathbf{GKJ})$, directly from (5).

ii: $\mathbf{E}$ is a basis for $\mathrm{range}((\mathbf{I} - \mathbf{G}_K^{+}\mathbf{G})\mathbf{KJ})$ (cf. (2)).

iii: $\mathbf{\Gamma}_r$ is a basis matrix for $\ker(\mathbf{J})$;

Hence, to prove that $\ker(\mathbf{Z}_C(0)^T) = \mathbf{0}$ it suffices to show that the raw spaces of the three blocks are also mutually linearly independent:

iv: The columns of the third block span $\ker(\mathbf{J})$, while the span of the columns of the first two blocks lies within $\mathrm{range}(\mathbf{J}^T)$;

v: $\mathrm{range}(\mathbf{G}_K^{+}\mathbf{\Gamma}_{uc})$ and $\mathrm{range}(\mathbf{I} - \mathbf{G}_K^{+}\mathbf{G})\mathbf{KE})$ are disjoint, then so are the spans of the columns of the first and second blocks. □

Note that the task–oriented priority order, in the choice of outputs, is reflected in the top–down ordering of outputs. In fact, the first group of outputs are coordinates for the subspace of rigid–body displacements of the manipulated object (in the basis $\mathbf{\Gamma}_{uc}$); similarly the second group of outputs for the subspace $\mathcal{F}_{hr}$ of active internal contact forces (in the basis $\mathbf{E}$), and the third group for the subspace of redundant degrees–of–freedom (in the basis $\mathbf{\Gamma}_r$). As a result of theorem 1, all of these three subspaces are functionally controllable, and so is their direct sum. In this sense, the chosen outputs provide a basis of the set of all functionally controllable outputs, that exactly corresponds to the task specifications introduced above.

5 Examples

Theorem 1 has been applied to the simple four planar devices reported in table 3. For each example, it is assumed that the manipulated object is a disk of unit radius, mass, and barycentral moment of inertia and that link masses with their distributions are such that the inertia matrix of the manipulator $\mathbf{M}_h(\cdot)$ is equal to the identity matrix. Moreover links are assumed to have unit length except for the link of the second finger in case 4 (its length is $3\cos(\pi/4)$). The length of a link involving contact with the object is meant to be measured between the joint axis and the contact point. Finally matrices $\mathbf{K}$, $\mathbf{B}$, $\mathbf{R}_q$ and $\mathbf{R}_{\dot{q}}$ are assumed to be normalized to the identity matrix.

Case 1: $\Gamma = \mathbf{0}$ and $E = [1\ 0\ -1\ 0]^T$. Being matrix Γ null, there are neither redundant motions for the manipulator nor rigid–body coordinate motions for the objects. The device can only apply force trajectories lying on $\mathsf{range}(\mathbf{E})$. According to theorem 1, the output matrix $\mathbf{C}$ has one row, namely $\mathbf{C} = [1\ |0\ 0\ 0\ |1\ |0\ 0\ 0\ ]$.

Case 2: In this case the manipulator has 2 joints, and at most two outputs can be functionally controllable. These can be specified according to the proposed method as one rigid–body coordinate motion of the object in the horizontal direction and one controllable internal forces:

$$\Gamma = \begin{bmatrix} \Gamma_{qc} \\ \Gamma_{uc} \end{bmatrix} = [1\ 1\ |1\ 0\ 0]^T; \quad E = [1\ 0\ -1\ 0]^T.$$

Then the output matrix results

$$\mathbf{C} = \left[\begin{array}{cc|ccc|cc|ccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & -1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{array}\right].$$

Case 3: The angle between the links is 30deg.

$$\Gamma = \begin{bmatrix} \Gamma_{qc} \\ \Gamma_{uc} \end{bmatrix} = \left[\begin{array}{ccc} 0 & 6.2 & 0 \\ 6.8 & -6.5 & 0 \\ 2.7 & 2.7 & 1 \\ 1 & 1 & -2.2 \\ \hline -7.6 & 7 & 0 \\ 1 & 1.2 & 1 \\ -3.4 & 1 & 1 \end{array}\right]; \quad E = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}.$$

Being $\mathbf{\Gamma}_{uc}$ f.r.r., it follows that the device can execute arbitrary object trajectories in $\mathbb{R}^3$ along with arbitrary internal contact forces trajectories ($\mathsf{range}(\mathbf{E}) = \mathsf{ker}(\mathbf{G})$).

$$\mathbf{C} = \left[\begin{array}{cccc|ccc|cccc|ccc} 0 & 0 & 0 & 0 & 0 & .2 & -.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .1 & .2 & -.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.1 & .5 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2.2 & 1 & -2.2 & -1 & 0 & 0 & 0 & 2.2 & 1 & -2.2 & -1 & 0 & 0 & 0 \end{array}\right],$$

Case 4: The manipulator is redundant and the angle between the consecutive links is equal to ± 90deg. The theorem output organization suggests to use two input degrees-of freedom to control rigid–body coordinate motion, one for internal contact forces ($\mathsf{range}(\mathbf{E})$) and the last one for redundancy. In fact

$$\Gamma = \begin{bmatrix} \Gamma_{qc} & \Gamma_{qc} \\ 0 & \Gamma_{uc} \end{bmatrix} = \left[\begin{array}{c|cc} 1 & -1 & 1 \\ -1 & 1 & 4.6 \\ -1 & -2 & -3.6 \\ 0 & -1 & 2.9 \\ \hline 0 & -2 & 6.6 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{array}\right]; \quad E = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix},$$

$$C = \left[\begin{array}{cccc|ccc|cccc|ccc} 0 & 0 & 0 & 0 & 1 & -1.3 & 2.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.5 & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 3 & 2 & 1 & -3 & 0 & 0 & 0 & 3 & 2 & 1 & -3 & 0 & 0 & 0 \\ \hline 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

6 Conclusions

This paper analyzes the problem of controlling motions of objects manipulated by kinematically defective mechanisms. Our main result consists in the suggestion of an organization of the vector of output variables for the dynamic system, that incorporate the constraints as well as the task requirements for the system. The approximate linearization method employed to study the problem renders our result valid only locally around an equilibrium point. The problem of generalizing this to the full nonlinear model is an interesting, if probably difficult, problem, especially in connection with the inclusion of rolling (nonholonomic) phenomena in the model.

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