Quasi-Static Compensation of Force Errors for Flexible Manipulators

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Abstract. This paper deals with the problem of controlling the interactions of flexible manipulators with their environment. For executing a force control task, a manipulator with intrinsic (mechanic) compliance has some advantages over a rigid manipulator. On the other hand, the deformations of the arm under the applied load give rise to errors in the evaluation of the kinematic model of the arm based on its rigid model. Since force feedback is usually provided by a force/torque sensor placed at the end-effector, kinematic errors reflect in force sensing errors, when a force task is specified in the world frame. This paper addresses the problem of evaluating these errors, and of compensating for them with suitable joint angle corrections. A solution to this problem is proposed in the simplifying assumptions that an accurate model of the arm flexibility is known, and that quasi-static corrections are of interest.

Keywords. Flexible Manipulators; Force Control; Nonlinear Optimization.

1. INTRODUCTION

Force control of manipulators in interaction with the environment poses a number of difficult problems, as abundantly illustrated in literature (a classical review is given by Whitney [1987]). Probably, the most important of these problems is the tendency of force control loops to become unstable in presence of small perturbations of the model parameters used to design the controller, due to the high stiffness of the manipulator-environment system. Reducing this stiffness certainly represents a viable way of alleviating force control problems: a more precise formulation of this concept can be found in [Roberts, Paul, and Hillberry, 1985], [Eppinger and Seering, 1987], and [Chiou and Shahinpoor, 1990]. This observation suggests an interesting application of flexible robots, that might encourage to design future robots with a built-in compliance. Besides this advantage, the design of lightweight, slender link robot arms meets other important requirements in applications such as space or underwater, and could reduce the cost of robot arms.

However, if the manipulator is composed of flexible links, and/or its joints are compliant, the positioning accuracy of the robot may result greatly reduced. The computation of the forward kinematics is affected by errors due to the deformations of its structure caused by contact forces. Also, assuming that the structural characteristics of the links are known, and that measures are available of its stress state (for example through strain gauges sensors), it is possible to determine the real position of the end effector, and to compute the applied force, i.e. to use the robot as a force sensor, see [Richter and Pfeiffer, 1991]. The goal of this paper is to compute the optimal joint position in order to minimize the error in a force-control task, given a local (wrist) force sensor and joint position information. This problem is basically regarded as a planning phase preceding real-time implementation, so that a quasi-static assumption is made, and dynamic effects of flexibility are disregarded.

A first difficulty arises from the fact that the force sensor is usually located as close as possible to the end-effector: the measured force is expressed in a reference frame whose position and orientation in the world frame depend in turn on the elastic deformations. A consequence is that the force information provided by the sensors can not be directly used in the force and position control loops. However, a recursive algorithm is presented to compute that position and orientation based on a model of the arm compliance, without resorting on any direct measure of the deformations. A substantially equivalent method has been presented by Fresonke, Hernandez, and Tesar [1988].

Once the deflections of the arm under the given load are known, appropriate corrections of the robot inputs can be applied to minimize the force/position errors due to flexibility. Since the overall deflection of the robot is comprised of both joint and link flexibilities, the determination of this correction is not trivial. In general, the modification that may be applied to the joint inputs will be able to compensate only in part for the deformation of the manipulator. The proposed method is framed in the context of nonlinear optimization, and may homogeneously consider both the case of defective and redundant manipulators.

The paper is organized as follows. In section 2, the model considered for the links of the flexible manipulator is described, and the main assumptions used in this paper explained. In section 3 the problem of the force error compensation is formulated as a minimization problem, and an algorithm for its solution is presented. Section 4 describes some specific details of the algorithm in the context under consideration, while section 5 reports some examples. The final section 6 concludes with comments about the suggested technique and plans for future activity.

2. MODEL OF A FLEXIBLE ARM

The general structure of a flexible arm dynamics can be described by a set of partial differential equations, of degree two for the torsional and axial modes, and four for the bending modes of the links. Consider for instance the simple manipulator link depicted in Fig. 1, with constant cross-section and lying in a horizontal plane. In this case, only the axial and bending deflections of the link in the plane are of interest. The axial and bending dynamics can be written as

$$EA\frac{\partial^2 x}{\partial \zeta^2} - \rho\frac{\partial^2 x}{\partial t^2} = 0 \qquad (1)$$

$$EJ\frac{\partial^4 y}{\partial \zeta^4} + \rho\frac{\partial^2 y}{\partial t^2} = 0 \qquad (2)$$

where ζ is the coordinate along the undeformed beam axis, $x = x(\zeta, t)$ is the displacement at time t of the section initially in ζ, E is the elastic modulus, ρ the mass per unit length, J is the cross-sectional moment of inertia, and y is defined as $y(\zeta, t) = w(\zeta, t) + \zeta q$ (see [Korolov and Chen, 1988]), being w the elastic displacement measured from the undeformed axis (see Fig. 1), and q the nominal (hub) position of the joint. Note that lumped joint elasticity is not considered in this example.

The overall manipulator dynamic relationship can be built (for serial link arms) by concatenating differential equations of this type by their boundary conditions. When only quasi-static conditions are considered, dynamical terms in the relationships above can be neglected. Therefore, eq.(1) and eq.(2) reduce to the following ordinary differential equations of elastic beams:

$$\frac{\partial^2 x}{\partial \zeta^2} = 0, \tag{3}$$

$$\frac{\partial^4 y}{\partial \zeta^4} = 0. \tag{4}$$

Deflections of links under the loads applied at their extremities can be determined by integrating such relationships. For instance, integrating eq.(3), eq.(4) with proper boundary conditions, one obtains:

$$\begin{aligned} \Delta x &= \frac{L \mathrm{f}_x}{EA}, \\ \Delta y &= \frac{L^3 \mathrm{f}_y}{3EJ} + \frac{L^2 \mathrm{m}}{2EJ}, \\ \Delta \theta &= \frac{L^2 \mathrm{f}_y}{2EJ} + \frac{L \mathrm{m}}{EJ}, \end{aligned}$$

where Δx, Δy are the displacements of the extremity of the link along the link axis and normal to it, respectively, and $\Delta\theta$ is the rotation of the link's end.

In the three-dimensional case, similar relationships hold, and a general linear equation relating the six elastic displacements of the link's end with the applied load can be written in matrix form as:

$$\begin{bmatrix} \Delta \mathbf{d} \\ \Delta \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \frac{1}{EA} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{L^3}{3EJ} & 0 & 0 & 0 & \frac{L^2}{2EJ} \\ 0 & 0 & \frac{L^3}{3EJ} & 0 & -\frac{L^2}{2EJ} & 0 \\ 0 & 0 & 0 & \frac{L}{GJ_o} & 0 & 0 \\ 0 & 0 & -\frac{L^2}{2EJ} & 0 & \frac{L}{EJ} & 0 \\ 0 & \frac{L^2}{2EJ} & 0 & 0 & 0 & \frac{L}{EJ} \end{bmatrix} \begin{bmatrix} \mathbf{f} \\ \mathbf{m} \end{bmatrix} \tag{5}$$

where $\Delta\mathbf{d}$ and $\Delta\boldsymbol{\theta}$ are the displacements and rotation vectors of the link's extremity in 3D space, G is the shear modulus of the link's material, and J_o is the polar momentum of the beam cross section.

The forces and torques on the distal ends of the links can be evaluated by means of static equilibrium considerations, using the recursive algorithm illustrated in section 4. The recursive calculation is started at the last link, where the load coincides with the robot - environment interaction forces, which are assumed to be known (by means e.g. of a wrist mounted force/torque sensor). However, this knowledge is relative to a reference frame E fixed to the end-effector. By solving the elastic displacements of every link, the geometric relationship of the end-effector frame E with the fixed (base) frame B can be found. Therefore, also the relationship between the interaction forces in E and in B is established as

$$\begin{bmatrix} {}^B\mathbf{f} \\ {}^B\mathbf{m} \end{bmatrix} = \begin{bmatrix} {}^B\mathbf{R}_E & \mathbf{0} \\ {}^B\mathbf{P}_{\otimes} {}^B\mathbf{R}_E & {}^B\mathbf{R}_E \end{bmatrix} \begin{bmatrix} {}^E\mathbf{f} \\ {}^E\mathbf{m} \end{bmatrix} \tag{6}$$

where ${}^B\mathbf{P}_{\otimes}$ is a skew-symmetric matrix equivalent to the cross product (${}^B\mathbf{p}\times$), being ${}^B\mathbf{p}$ the vector of the linear displacements of the end-effector caused by both the flexibility and the kinematics of the mechanism, and ${}^B\mathbf{R}_E$ is the corresponding rotation matrix.

Introducing the symbol ${}^B\mathbf{K}_E$ for the force tranformation matrix, and the vector $\mathbf{w} = \left[\mathbf{f}^T\ \mathbf{m}^T\right]^T$ (*wrench*), we rewrite for convenience eq.(6) as

$${}^B\mathbf{w} = {}^B\mathbf{K}_E\, {}^E\mathbf{w}. \tag{7}$$

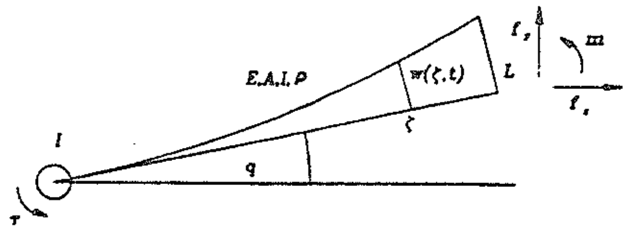

Figure 1: A simple manipulator link and its elastic model

3. COMPENSATION OF THE FORCE ERRORS

As mentioned in the introduction, our goal is to compensate for force errors during task execution, i.e. to compute and apply proper set-points to the position/force controllers of the manipulator's joints to minimize these errors.

The correction of interaction force errors may be cast in a nonlinear optimization problem form. The kernel of the adopted algorithm is related to the steepest descent method [Press et al., 1986], and it has been utilized in other fields of robotics, in particular in the solution of the inverse kinematic problem for redundant and non-redundant manipulators, see for instance [Balestrino, DeMaria and Sciavicco, 1984], [Wolovich and Elliott, 1984], [Sciavicco and Siciliano, 1986, 1988], [Das, Slotine and Sheridan, 1988].

The basic idea is the following. Define a wrench error as

$$\mathbf{e} = \begin{bmatrix} {}^B\mathbf{f}_d \\ {}^B\mathbf{m}_d \end{bmatrix} - \begin{bmatrix} {}^B\mathbf{f} \\ {}^B\mathbf{m} \end{bmatrix} = {}^B\mathbf{w}_d - {}^B\mathbf{w} \tag{8}$$

and a quadratic positive definite function as

$$V(\mathbf{e}) = \frac{\mathbf{e}^T\mathbf{P}\mathbf{e}}{2} \tag{9}$$

where $\mathbf{P}$ is a symmetric positive definite matrix. Because of equations (6)-(7), the wrench ${}^B\mathbf{w}$ is a function of $\mathbf{q}$, so that $V(\mathbf{e})$ depends on the joint position vector. The optimal joint positions $\mathbf{q}$ are those minimizing $V(\mathbf{q})$, and therefore we seek a control law for the arm joint positions such that the robot is driven towards the optimal configuration. Since $V(\mathbf{q})$ may be regarded as a Lyapunov function, the convergence to its minimum is guaranteed if the position control law is such that the value of V is kept decreasing along the trajectories of the system.

In the continuous time domain, the convergence to the minimum is achieved if the following Lyapunov condition is satisfied:

$$\dot{V} \doteq \mathbf{e}^T\mathbf{P}\dot{\mathbf{e}} < 0. \tag{10}$$

where

$$\dot{\mathbf{e}} = {}^B\dot{\mathbf{w}}_d - \frac{\partial({}^B\mathbf{K}_E\, {}^E\mathbf{w})}{\partial t} = {}^B\dot{\mathbf{w}}_d - \mathbf{G}\dot{\mathbf{q}} \tag{11}$$

being $\mathbf{G} = \frac{\partial({}^B\mathbf{K}_E\, {}^E\mathbf{w})}{\partial \mathbf{q}}$ the Jacobian matrix of ${}^B\mathbf{w}$. If the update law for the joint position is chosen as

$$\dot{\mathbf{q}} = \lambda + [\frac{(\mathbf{e}^T\mathbf{P}\,{}^B\dot{\mathbf{w}}_d)}{(\mathbf{e}^T\mathbf{P}\mathbf{G}\mathbf{G}^T\mathbf{P}\mathbf{e})}]\mathbf{G}^T\,\mathbf{P}\mathbf{e},\ \ \lambda > 0, \tag{12}$$

it can be easily shown that the condition (10) is verified. Sciavicco and Siciliano [1988], using the same algorithm for solving the inverse kinematic problem, pointed out how eq.(12) may be, for computational convenience, simplified to

$$\dot{\mathbf{q}} = \lambda\mathbf{G}^T\,\mathbf{P}\mathbf{e}, \tag{13}$$

allowing in this case the function $\dot{V}$ to be negative-definite only outside a region of the error space containing the stability point $\mathbf{e} = 0$. The resulting algorithm is shown as a block diagram in Fig. 2.

In discrete time, the stability proof of the algorithm is more complex. A detailed discussion of such proof has been presented

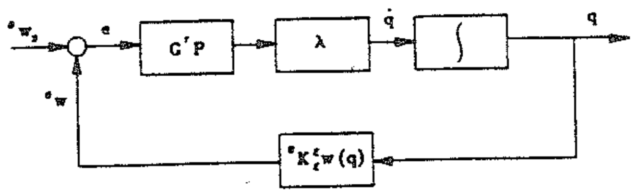

Figure 2: Block diagram of the proposed algorithm.

in [Das, Slotine, and Sheridan, 1988] for the inverse kinematic solution of redundant manipulators. One of the major modifications in discrete time is that, in order both to obtain the maximum convergence rate for the scheme and to avoid instability problems, the gain λ has to be updated at each sampling period T. In fact, given the discrete time version of the Lyapunov function, eq.(9), at $t = nT$, V_n, the goal is to make negative the difference $V_{n+1} - V_n$. If the joint velocities are computed at $t = nT$ as

$$\dot{\mathbf{q}}_n = \lambda_n \mathbf{G}_n^T \mathbf{P} \mathbf{e}_n, \tag{14}$$

the convergence of the algorithm is guaranteed with the choice

$$\lambda_n = \frac{1}{T} \frac{\mathbf{e}_n^T \mathbf{P}^T \mathbf{G}_n \mathbf{S}_n \mathbf{G}_n^T \mathbf{P} \mathbf{e}_n}{\mathbf{e}_n^T \mathbf{P}^T \mathbf{G}_n \mathbf{S}_n \mathbf{G}_n^T \mathbf{P} \mathbf{G}_n \mathbf{S}_n \mathbf{G}_n^T \mathbf{P} \mathbf{e}_n} \tag{15}$$

where $\mathbf{S}_n$ is a diagonal matrix whose elements are properly computed to limit the maximum values of $\mathbf{G}_n^T \mathbf{P} \mathbf{e}_n$, [Das, Slotine, and Sheridan, 1988].

4. COMPUTATION OF ${}^{B}\mathbf{K}_E$ AND G

In order to apply the algorithm, it is necessary to compute the matrices ${}^{B}\mathbf{K}_E$ and $\mathbf{G} = \frac{\partial({}^{B}\mathbf{K}_E\,{}^{E}\mathbf{w})}{\partial \mathbf{q}}$, eq.(7), (11). In this section, these two matrices are calculated for a manipulator with a generic kinematic structure with n degrees-of-freedom.

Computation of ${}^{B}\mathbf{K}_E$. The wrench transformation matrix, see eq.(6), is defined as

$${}^{B}\mathbf{K}_E = \begin{bmatrix} {}^{B}\mathbf{R}_E & \mathbf{0} \\ {}^{B}\mathbf{P}_{\otimes}\,{}^{B}\mathbf{R}_E & {}^{B}\mathbf{R}_E \end{bmatrix} \tag{16}$$

The sub-matrices ${}^{B}\mathbf{R}_E$ and ${}^{B}\mathbf{P}_{\otimes}$ are in general complex functions of the compliance of the joints, of the link flexibility and of the kinematics of the manipulator. Since these quantities depend on the interaction forces and the joint position, in general it is not trivial to compute ${}^{B}\mathbf{K}_E$. A possible and efficient way to determine this matrix is to use a recursive method, consisting in computing the effects of the force/torque applied to the i-th link on the (i-1)-th link, starting with the distal link. These effects are expressed by a transformation matrix $\mathbf{K}_i$, with the same structure as in eq.(16), and such that

$$\mathbf{w}_{i-1} = \mathbf{K}_i \mathbf{w}_i \tag{17}$$

Therefore, the matrix ${}^{B}\mathbf{K}_E$ can be expressed as the product of the $\mathbf{K}_i$ as

$${}^{B}\mathbf{K}_E = \mathbf{K}_1 \mathbf{K}_2 \ldots \mathbf{K}_n = \prod_{i=1}^{n} \mathbf{K}_i \tag{18}$$

where the generic term $\mathbf{K}_i$, according to the hypothesis of small deformations, can be computed as

$$\mathbf{K}_i = \mathbf{K}_{i,kin}(q_i)\mathbf{K}_{i,joint}(\tau_i)\mathbf{K}_{i,flex}(\mathbf{w}_i) \tag{19}$$

where $\mathbf{K}_{i,kin}$ depends on the geometry of the link, $\mathbf{K}_{i,joint}$ on the joint compliance, and $\mathbf{K}_{i,flex}$ on the link flexibility. In the following, the expressions of these three transformation matrices are given. For the sake of simplicity, in the following discussion only rotational joints are taken into account in the kinematic structure of the manipulator: however, similar considerations may be done also for the case of prismatic joints.

The elements of the matrices $\mathbf{R}_{i,kin}$ and $\mathbf{P}_{\otimes,i,kin}$ are only functions of the geometric and kinematic parameters of the i-th link. Using the standard Denavit-Hartenberger notation for the description of the kinematic parameters, the expression of $\mathbf{R}_{i,kin}$ is the standard rotation matrix about the joint axis, and $\mathbf{P}_{\otimes,i,kin}$ may be composed from the elements of the vector
$\mathbf{p}_{i,kin} = [a_i \cos(q_{i-1}) + \sin(q_{i-1}) \sin(\alpha_i) d_i$
$a_i \sin(q_{i-1}) - \cos(q_{i-1}) \sin(\alpha_i) d_i \quad \cos(\alpha) d_i]^T$.

The matrix $\mathbf{K}_{i,joint}$ is composed by the sub-matrices $\mathbf{R}_{i,joint}$ and $\mathbf{P}_{\otimes,i,joint}$, where $\mathbf{R}_{i,joint} = Rot(z_i, k_{ci}\tau_i)$ is a rotation matrix about the joint (z) axis, and $\mathbf{P}_{\otimes,i,joint}$ is composed with the elements of the vector $\begin{bmatrix} -a_i[1-\cos(k_{c,i}\tau_i)] & a_i \sin(k_{c,i}\tau_i) & 0 \end{bmatrix}^T$. $k_{c,i}$ and τ_i are the joint compliance and the torque acting on the joint, respectively.

Finally, the i-th matrix $\mathbf{K}_{i,flex}$ is composed by the submatrices $\mathbf{R}_{i,flex}$ and $\mathbf{P}_{\otimes,i,flex}$, whose elements are functions of the elastic displacements of the link. The evaluation of such displacements is not an easy task in general. Sophisticated numerical techniques, such as FEM, can be applied profitably to this problem; in other cases, a direct calibration of displacements under known loads can be a viable solution to obtain a flexibility model for the arm. For our purposes here, however, a rather simple flexibility model for the links, considered as slender beams with constant section, can suffice. In the hypothesis of small elastic deformations, the relation between the 6-dimensional vector of linear and rotational displacements of the link $\begin{bmatrix} \Delta \mathbf{d}_{i,flex}^T & \Delta \theta_{i,flex}^T \end{bmatrix}^T$ and the applied force $\mathbf{f}_i$ and torque $\mathbf{m}_i$ is reported in section 2, eq.(5). The matrix $\mathbf{R}_{i,flex}$ is computed as a rotation matrix about an axis parallel to $\Delta\theta_{i,flex}$ of an angle given by the module of $\Delta\theta_{i,flex}$. As pointed out in [Fresonke, Hernandez and Tesar, 1988], this approximation is valid as long as small deformations are assumed.

The recursive computation of eq.(17), starting from the distal joint for which the force/torque sensor provides ${}^{E}\mathbf{w}$, yields the deformations generated by the flexibility and the joint compliance, and therefore all the matrices in eq.(18), allowing the transformation of the force vector from the frame E located in the end-effector to the base frame B.

Computation of $\mathbf{G} = \frac{\partial {}^{B}\mathbf{w}}{\partial \mathbf{q}}$. The application of the algorithm presented in section 3 requires the computation of the matrix $\mathbf{G} = \frac{\partial {}^{B}\mathbf{w}}{\partial \mathbf{q}} = \frac{\partial({}^{B}\mathbf{K}_E\,{}^{E}\mathbf{w})}{\partial \mathbf{q}}$, in which, in general, both ${}^{B}\mathbf{K}_E$ and ${}^{E}\mathbf{w}$ are functions of the joint position vector $\mathbf{q}$. Since the wrench transformation matrix ${}^{B}\mathbf{K}_E$ is a function of the joint positions, see equations (6), (7), the Jacobian matrix G may be computed as

$$\mathbf{G} = \frac{\partial({}^{B}\mathbf{K}_E\,{}^{E}\mathbf{w})}{\partial \mathbf{q}} = \frac{\partial(\prod_{i=1}^{n} \mathbf{K}_i\,{}^{E}\mathbf{w})}{\partial \mathbf{q}}$$

$$= \frac{\partial(\prod_{i=1}^{n} \mathbf{K}_i)}{\partial \mathbf{q}}\,{}^{E}\mathbf{w} + (\prod_{i=1}^{n} \mathbf{K}_i)\frac{\partial {}^{E}\mathbf{w}}{\partial \mathbf{q}}.$$

In general, the computation of this matrix requires the knowledge of the manipulator-environment interaction, eq.(5), for the calculation of $\frac{\partial {}^{E}\mathbf{w}}{\partial \mathbf{q}}$. In the following, only the case in which ${}^{E}\mathbf{w}$ may be considered constant in the range of motion caused by elastic displacements is taken into account. The matrix G may be computed as

$$\mathbf{G} \approx \frac{\partial(\prod_{i=1}^{n} \mathbf{K}_i)}{\partial \mathbf{q}}\,{}^{E}\mathbf{w} = \mathbf{T}\,{}^{E}\mathbf{w}$$

where T is a three-dimensional matrix composed by n $\mathbf{T}_i$ (6x6) matrices given by

$$\mathbf{T}_i = (\prod_{j=1}^{i-1} \mathbf{K}_{j,kin}\mathbf{K}_{j,joint}\mathbf{K}_{j,flex})\frac{\partial \mathbf{K}_{j,kin}}{\partial q_i}(\prod_{j=i}^{n} \mathbf{K}_{j,joint}\mathbf{K}_{j,flex}\mathbf{K}_{j+1,kin})$$

where conventionally we assume $\mathbf{K}_{n+1,kin} = \mathbf{I}_6$.

Therefore, the generic i-th column of the matrix **G** is calculated as

$$\mathbf{G}_i = \mathbf{T}_i\,{}^E\mathbf{w}$$

5. CASE STUDIES

In this section, two examples of the application of the algorithm are illustrated. The case studies refer to the simulation of two planar manipulators with rotational joints. The manipulators are a planar 3 degrees-of-freedom and a redundant 5 degrees-of-freedom planar arm. Although the above reported formulas are applicable to generic links, for simplicity here all the links of both manipulators have the same geometrical and mechanical structure, with a circular cross-section with radius 10 mm, and a length of 1000 mm. The material elastic constants are those of common steel, while for the joints a compliance of $k_{ci} = 10^3$ (Nm/rad) has been assumed. The matrix **P** is, in both the examples, an identity matrix with proper dimensional units.

A 3 Degrees-of-Freedom Planar Manipulator. The manipulator considered in the first example consists of three links and three rotational joints with parallel axes. A wrench ${}^E\mathbf{w} = \begin{bmatrix} 0 & 10 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ (N)-(Nm) is applied to the tip of the arm. The initial (undeformed) pose of the manipulator is shown in Fig. 3.a. The effects of the applied wrench on the flexible-compliant structure are reported in Fig. 3.b. Since the joint encoders are placed before the elastic elements, the arm appears to the control system to be in the undeformed configuration of Fig. 3.a. In this situation, if the force set-point ${}^B\mathbf{w}_d = \begin{bmatrix} 0 & 10 & 0 & 0 & 0 & 30 \end{bmatrix}^T$ (N)-(Nm) is specified, a force error ${}^B\mathbf{e} = \begin{bmatrix} 1.1561 & 0.067 & 0 & 0 & 0 & 0.02506 \end{bmatrix}^T$ (N)-(Nm) results. In Fig. 3.c the computed corrective actions on the joint positions are shown, i.e. the nominal configuration to which the arm has to be positioned to minimize the force error **e**. In Fig. 3.d the final (compensated) configuration of the arm is shown. A final value of the error function $V(\mathbf{e}) \leq 10^{-6}$ is achieved in 335 steps.

A 5 Degrees-of-Freedom Planar Redundant Manipulator. The second manipulator taken into consideration is a redundant planar arm. The robot, shown in Fig. 4.a, is a 5 degrees-of-freedom robot. A wrench ${}^E\mathbf{w} = \begin{bmatrix} 0 & 10 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ (N)-(Nm) is supposed applied to the tip of the arm, as in the previuos case. In this case, the force error, with a set point ${}^B\mathbf{w}_d = \begin{bmatrix} 0 & 10 & 0 & 0 & 0 & 50 \end{bmatrix}^T$ (N)-(Nm), is ${}^B\mathbf{e} = \begin{bmatrix} 2.29975 & 0.4598 & 0 & 0 & 0 & 0.3656 \end{bmatrix}^T$ (N)-(Nm). In Fig. 4.a-d the initial undeformed and deformed configurations, as well as the corrective actions and the final position of the manipulator are shown. In this case, because of the larger number of joints and the greater value of the initial error, a value of the error function $V(\mathbf{e}) = 9.1\ 10^{-6}$ is reached after 1427 iterations of the algorithm.

6. CONCLUSIONS

In this paper, an algorithm for the compensation of interaction force errors for flexible manipulators has been presented. Because of the joint compliance and the link flexibility this problem is non-linear, and the proposed solution is framed in the context of non-linear optimization. In fact, the presented algorithm may be related to the steepest descent method, a well-known technique in this area. The basic idea is to control the joint positions to minimize a quadratic function the force error. The choice for the joint position updating law results in an algorithm similar to algorithms proposed for the kinematic solution of redundant manipulators. With this choice, it is possible to demonstrate the stability and convergence of the algorithm with a Lyapunov stability analysis. Examples of the proposed algorithm are reported, showing the effectiveness of the technique also in the case of redundant arms. The proposed solution is used in this paper as an off-line reference generator. However, it is authors' opinion that, with proper modifications, it may be applied also as an on-line (real-time) high-level control level for the force control of flexible manipulators.

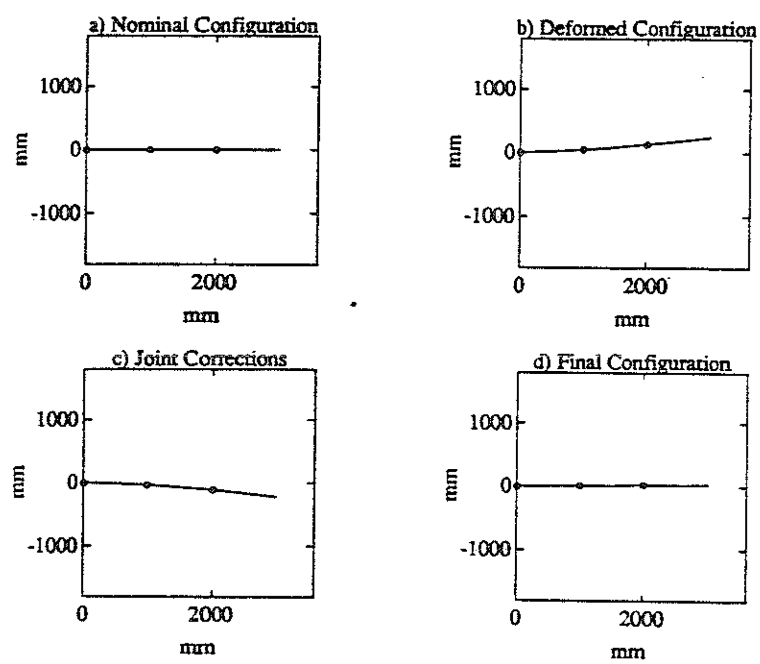

Fig. 3. A 3 dof planar manipulator: a) nominal configuration; b) deformed initial configuration; c) computed corrections; d) final compensated configuration.

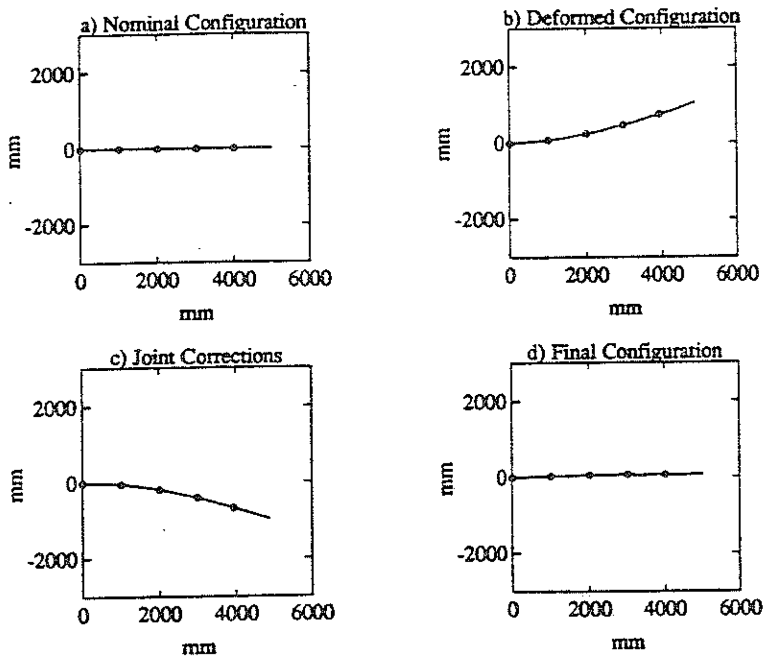

Fig. 4. A 5 dof planar manipulator: a) nominal configuration; b) deformed initial configuration; c) computed corrections; d) final compensated configuration.

Further developments of this research will investigate extensions of the method to more general problems. In particular, activity is currently in progress in the following areas:

- modification of the method to take into account interaction forces that vary (in the end-effector reference frame) during the execution of the task, such as e.g. gravity and inertial loading;
- modification of the iterative algorithm in order to achieve better convergence speed and trajectory smoothness, so as to allow the application of the method as an on-line (real-time) high-level force controller for flexible manipulators.

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REFERENCES

Balestrino, A., De Maria, G., Sciavicco, L., "Robust Control of Robotic Manipulators", Proc. 9th. IFAC World Congress, Budapest, Hungary, 1984.

Chiou, B.C., and Shahinpoor, M, "Stability Considerations for a two Link Force Controlled Flexible Manipulator", Proc. IEEE Int. Conf. on Robotics and Automation, Cincinnati, OH, May 1990.

Das, H., Slotine, J-J.E., Sheridan, T.B., "Inverse Kinematic Algorithms for Redundant Systems", Proc. IEEE Int. Conf. on Robotics and Automation, San Francisco, CA, 1988.

Eppinger, S.D., and Seering, W.P., "Understanding Bandwidth Limitations in Robot Force Control", Proc. IEEE Int. Conf. on Robotics and Automation, Raleigh, NC, March 1987.

Fresonke, D.A., Hernandez, E., Tesar, D., "Deflection Prediction for Serial Manipulator", Proc. IEEE Int. Conf. on Robotics and Automation, Philadelphia, PA, 1988.

Korolov, V.V., and Chen, Y.H.: "Robust Control of a Flexible Manipulator Arm", Proc. IEEE Int. Conf. Robotics and Automation, Philadelphia, PA, 1988.

Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T., "Numerical Recipes: The Art of Scientific Computing", Cambridge University Press, 1986.

Richter, K., Pfeiffer, F., "A Flexible Link Manipulator as a Force Measuring and Controlling Unit", Proc. IEEE Conf. Robotics and Automation, Sacramento, CA, 1991.

Roberts, R.K., Paul, R.P., and Hillberry, B.M., "The Effect of Wrist Sensor Stiffness on the Control of Robot Manipulators", Proc IEEE Int. Conf. on Robotics and Automation, St. Louis, March 1985.

Sciavicco, L., Siciliano, B., "Coordinate Transformation: A Solution Algorithm for One Class of Robots", IEEE Trans. Syst., Man, Cybern., Vol. SMC-16, No. 4, July/Aug. 1986.

Sciavicco, L., Siciliano, B., "A Solution Algorithm to the Inverse Kinematic Problem for Redundant Manipulator", IEEE Jour. of Robotics and Automation, Vol. 4, No. 4, Aug. 1988.

Whitney, D.E., "Historical Prespective and State of the Art in Robot Force Control", Int. Jour. of Robotic Research, Vol. 6, No. 1, Spring 1987.

Wolovich, W.A., Elliott, H., "A Computational Technique for Inverse Kinematics", Proc. 23rd. Conf. on Decision and Control, Las Vegas, NV, Dec. 1984.